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A Review of the Lynden-Bell/Choloniewski Method for Obtaining Galaxy Luminosity Functions

Part II

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Abstract

In order for cosmologists to discern the large-scale structure of the cosmos, it is necessary to determine the true distribution of galaxies in space. In order to do this, however, it is necessary to correct for the fact that some dim galaxies are too faint to be seen (the *Malmquist bias*). This correction is often obtained via a *luminosity function*, which gives the number density of galaxies (galaxies per unit of comoving volume) per bin of intrinsic brightness (or absolute magnitude bin). This review uses real data from the Sloan Digital Sky Survey to demonstrate one such method for obtaining the luminosity function, the Lynden-Bell/Choloniewski method.

Introduction

A detailed review of the theory behind the Lynden-Bell/Choloniewski (LBC) method was presented by Hebert and Lisle (2016). In the LBC method, one plots the distance moduli μ of the N_{obs} visible galaxies within a survey against the absolute magnitudes M for those same galaxies (a demonstration plot for a "survey" of galaxies with $N_{obs} = 10$ is shown in Figure 1). It should be noted that prior to using the method, each galaxy absolute magnitude should be K-corrected and each galaxy distance modulus μ should be replaced by $\mu + K_{avg}(z)$. Although this discussion follows the notation of Choloniewski (1987), it should be noted that neither Lynden-Bell (1971) nor Choloniewski (1987) explicitly mentioned the need to perform the K-correction in their papers.

Choloniewski (1987) showed that the number density of observed galaxies in μ-M space (actually, $[\mu + K_{avg(z)}]$-$M_{corrected}$ space, to be more precise) may be expressed as

$$\sum_{k=1}^{N_{obs}} \delta(M - M_k, \mu - \mu_k) =$$

$$\sum_{i:}^{M_i+\mu_j \leq m_{max}} \sum_{j:} \psi_i d_j \delta(M - M_i)\delta(\mu - \mu_j) \quad (1)$$

The weighting ψ and d factors may be obtained by integrating Eq. (1) subject to judicious choices for the limits of

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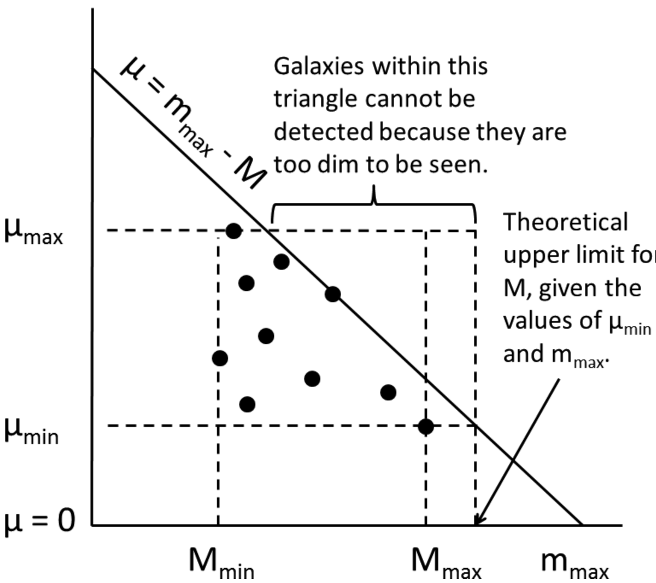

Figure 1. Plot of distance modulus μ versus absolute magnitude M for a simulated magnitude-limited survey containing ten observable galaxies. Of course, a real galaxy survey may contain many thousands of such galaxies. Note that the diagonal line is defined by the limiting apparent magnitude m_{max} of the survey, such that no galaxies are observed within the triangle above the diagonal line. Complications regarding m_{max} and M_{max} that are encountered in real galaxy surveys are discussed in the text.

integration, as discussed by Choloniewski (1987) and Hebert and Lisle (2016). Once these weighting factors have been obtained, the luminosity function is given by

$$\phi(M) = \frac{1}{V_t}\left[\sum_{i=1}^{N_{obs}} \psi_i \delta(M - M_i)\right]\left[\sum_{j=1}^{N_{obs}} d_j\right] \qquad (2)$$

where V_t is the total comoving volume of the galaxy survey.

The Sloan Digital Sky Survey

The Sloan Digital Sky Survey (http://www.sdss.org/) is a major survey of celestial objects that covers about 35% of the sky. The survey utilizes a 2.5 meter wide-field telescope at the Apache Point Observatory in New Mexico (York et al., 2000, p. 1580). This telescope is equipped with a CCD camera that can image the sky in five optical bands, as well as two digital spectrographs. In addition to this 2.5-meter telescope, two other telescopes were used in the process of calibrating photometric magnitudes, a USNO 40-inch telescope and an SDSS photometric telescope (PT). More than 900 million unique objects (stars, galaxies, and quasars) have been detected at the time of the tenth data release in July 2013. Spectroscopic measurements have been made for 859,322 unique galaxies, permitting redshifts to be calculated for those galaxies (http://www.sdss3.org/dr10/scope.php#opticalstats). Helpful websites for downloading Sloan data are http://skyserver.sdss3.org/dr9/en/help/docs/default.asp and http://skyserver.sdss3.org/casjobs/.

The SDSS photometric system consists of five broad color bands, denoted as u, g, r, i, and z, which are designed to facilitate the detection of faint objects and provide coverage of the whole accessible range of optical wavelengths (3000 Å to 11, 000 Å).

SDSS Apparent Magnitudes

There are several different ways to estimate the apparent magnitude of a galaxy in a given filter. Hence, one question that must be resolved before analyzing patterns in galaxy spatial distribution is this: Which is the best apparent magnitude system to use for such an analysis?

The standard definition of apparent magnitude (or Pogson apparent magnitude) is given in terms of the flux (photons per unit area) within a specified wavelength range of a star or galaxy by the following formula:

$$m_P = -2.5\log_{10}\left(\frac{f}{f_0}\right) \qquad (3)$$

where f_0 is a conventionally agreed-upon reference flux. The reference flux sets the zero point of the magnitude system and corresponds closely to the brightness of the star Vega. Though the apparent magnitude is easily estimated for stars, there is a problem with using this definition for galaxy surveys.

Because galaxies are extended objects, rather than point sources, there are complications in attempting to determine their total flux within a particular band (such as their lack of well-defined “edges”). The apparent magnitude of a galaxy is defined to be that which it would have if all of its light were concentrated into a point. This is perfectly well defined in principle. But in practice, this can be very difficult to estimate with high precision because the sky itself is not totally dark and it is difficult to disentangle sky flux from galaxy flux, particularly near the dark and ill-defined limb (edge) of the galaxy. So, a number of different methods have been developed to give a reasonable and pragmatically workable estimation of a galaxy’s apparent magnitude.

One is by calculating the galaxy’s so-called Petrosian flux (Petrosian, 1976). The Petrosian flux and magnitude systems were designed to measure a consistent fraction of the galaxy's total light, regardless of the galaxy's distance and location. This is achieved by measuring galaxy fluxes within a circular aperture whose radius is determined by the shape of the (azimuthally-averaged) surface brightness profile *I (r)*, which is the brightness for a standardized portion of the extended object. Surface brightness is usually measured in magnitudes per square arc second. Because radiative flux decreases with the square of the distance, while surface area increases with the square of the distance, the surface brightness is independent of distance. The SDSS survey uses a modified version of the Petrosian flux as described at http://ned.ipac.caltech.edu/help/sdss/dr6/photometry.html#mag_petro.

A second way is via the use of luminosity profiles. If one can fit a mathematical luminosity or brightness function, which describes the way in which the galaxy's (azimuthally averaged) brightness varies with radial distance from the galaxy's center, then one can obtain the total brightness (or equivalently, the total flux) of that galaxy by integrating this function over all possible brightnesses or luminosities (i.e., from zero to infinity). The radially averaged brightness of an elliptical galaxy (as a function of apparent distance *r* from its center) is given by a *de Vaucouleurs profile*:

$$I_{ellipse}(r) = \mathrm{I}_e \exp^{-7.669\left[\left(\frac{r}{re}\right)^{1/4}-1\right]} \quad (4)$$

where r_e is the radius of the galaxy's inner disk contributing half the galaxy's brightness and I_e is the brightness at that radius. The brightness of the disk of a spiral or lenticular galaxy is described by a decaying exponential:

$$I_{disk}(r) = I_0 \exp(-r / r_e) \quad (5)$$

Note that these two equations assume that the galaxy's inclination to the observer has already been taken into account when obtaining these brightness functions.

Since spiral galaxies often consist of an ellipsoidal "bulge," as well as a disk, the composite flux of a spiral galaxy can be modeled as a superposition of the de Vaucouleurs and exponential brightness profiles. Moreover, such a superposition can be used, without loss of generality, for galaxies that are primarily elliptical or that are disklike and yet lack a prominent central bulge. The composite flux of a galaxy was thus defined by the SDSS team to be:

$$f_{composite} = fracDeVf_{deV} + (1 - fracDeV)\, f_{exp} \quad (6)$$

Where f_{deV} and f_{exp} are the de Vaucouleur and exponential fluxes, respectively, and *fracDeV* is the fraction of this composite flux due to the de Vaucouleur profile. Although one could conceivably generate more complicated models than this for certain extended objects, the SDSS team opted against this on the basis that such additional computational expense was not warranted for the bulk of detected objects.

Once the flux has been obtained via one of these methods, it may be used to obtain the galaxy's apparent magnitude *m* for a particular band. However, as noted earlier, there is a complication in attempting to use the traditional Pogson definition of apparent magnitude: calculated apparent magnitudes are subject to very large errors when *f* is comparable to the flux of the background sky. Because many galaxies are quite dim, this is a problem for galaxy surveys. In order to circumvent this problem, Lupton, Gunn, and Szalay (1999) devised a modified magnitude system based upon the inverse hyperbolic sine function. These "asinh" magnitudes are defined by

$$m_{asinh} = -\left[2.5 / \ln(10)\right]\left[\frac{asinh(f / f_0)}{2b} + \ln b\right] \quad (7)$$

where b is a "softening parameter" designed to minimize the noise in values of m. The value of b for the r-band is 1.2×10^{-10} (http://classic.sdss.org/dr7/algorithms/fluxcal.html#asinh_table). This is the apparent magnitude definition used by the SDSS survey after obtaining the galaxy flux (http://classic.sdss.org/dr7/algorithms/photometry.html#mag_psf) via the methods discussed above.

Once this flux was obtained, it was substituted into Eq. (7) in order to obtain the apparent magnitude. So-called *cmodel* magnitudes were calculated using a flux that was a best-fit superposition of the de Vaucouleurs and exponential profiles. A similar *model* magnitude was obtained by choosing the better fit of the de Vaucouleurs and exponential profiles to obtain the flux, rather than a superposition of the two.

The optimal magnitude system depends upon the task being performed, but the SDSS team noted that the cmodel apparent magnitude seems to be the best overall choice due to "close to optimal noise properties" that make it an even better choice than Petrosian magnitudes (http://classic.sdss.org/dr7/algorithms/photometry.html#mag_psf). However, the SDSS team noted that *model* magnitudes are the better choice for measuring the colors of extended objects such as galaxies.

In any case, the differences between the apparent magnitude systems are expected to be small. For instance, differences between cmodel and Petrosian magnitudes are only about 0.05–0.1 mag for bright galaxies.

Target Selection in the Main Galaxy Sample

This demonstration of the LBC method utilizes galaxies from the flux-limited main galaxy sample. The following is a brief summary of the main sample selection criteria provided in Strauss et al. (2002).

In order to simplify the selection process, only one limiting apparent (Petrosian) magnitude was used, rather than five. The SDSS team reasoned that this band should be either the *r* or *i* band, for the following reasons:

- *K*-corrections in an *r* (red) band or an *i* (far red) band tend to be smaller than those for other bandpasses.
- Galaxy fluxes are thought to be dominated by red stars.
- Inferred absolute magnitudes for the *i* and *r* bands are less affected by the phenomenon of galactic reddening than for other bands.

The *r* band was finally selected over the *i* band since the sky background is dimmer and less variable in the *r*-band, thus providing a better contrast to enable easier detection of galaxies against the sky background. The method of Schlegel, Finkbeiner, and Davis (1998) was used to correct for galactic extinction, the absorption and scattering of electromagnetic radiation due to the presence of galactic dust. After making this correction, Petrosian *r* magnitudes were calculated. Then saturated, bright, or blended objects were rejected, and the limiting apparent magnitude was set at $m_{\text{lim, Petrosian}} = 17.77$. This magnitude cutoff was chosen in order to obtain an average of at least 90 galaxies per square degree, as this angular galaxy density is believed to correspond to the depth at which galaxy number can vary substantially due to large-scale structure. Hence, this magnitude cutoff should facilitate the detection of large-scale galaxy structure that may exist.

However, surface brightness selection criteria were also imposed on the survey. Galaxies with a Petrosian half-width surface brightness (defined to be the average surface brightness within a circular radius containing half the Petrosian flux) less than or equal to 23.0 mag arcsecs^{-2} were included, although dimmer galaxies (higher surface brightness values) might also be included if they met other additional criteria. These surface brightness criteria were needed in order to ensure reliable spectroscopic results, which are necessary for an accurate determination of redshift. A schematic illustrating the selection process is provided in Figure 5 of Strauss et al (2002).

Completeness of a survey is defined to be the fraction of observed true galaxies that have been correctly identified as such (Ball and Brunner, 2010). By comparing galaxies identified by the SDSS survey to galaxies identified in the Zwicky catalog, the SDSS team estimated that the main galaxy sample had an overall completeness of more than 99% but that this completeness dropped to 95% for brighter galaxies, due to "blending" with saturated stars. Note that the concept of completeness refers to the percentage of correctly *identified* objects, *not* the percentage of objects that have been detected! In other words, even with a survey completeness of 99%, it is quite possible that the percentage of detected galaxies (compared to the true number of galaxies within the bounds of the survey) could still be quite small due to Malmquist bias. Hence, even a completeness of 99% does not remove the need for a luminosity function and selection function. The SDSS team estimated that nearly all (99.9%) spectroscopically observed galaxies in the main sample yielded successful redshift determinations. However, they estimated that 6% of galaxies were missed because of close angular proximity to a companion galaxy but that this could be accounted for by appropriate weighting of closely paired galaxies. Reproducibility of the sample was estimated at 94.5%.

A chart showing the galaxies in the main survey as a function of right ascension (RA) and declination (dec) is provided in Figure 2. Note that these celestial coordinates are given in the J2000.0 epoch (York et al., 2000, p. 1580; Hilton and Hohenkerk, 2004). The main ellipse of the main survey was chosen to maximize observing efficiency and to minimize the effects of galactic extinction (Strauss et al. 2002, p. 1811).

The Schechter Function

Galaxy luminosity functions are generally well characterized by a *Schechter function* (Schechter, 1976). Expressed in terms of absolute magnitude M, the number of galaxies per unit co-moving volume per absolute magnitude interval dM is given by

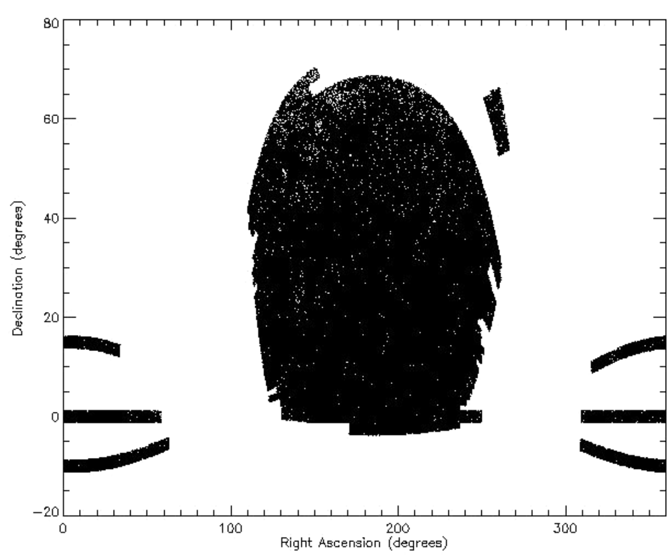

Figure 2. Sky plot of galaxies within the main galaxy portion of the SDSS survey (10th data release).

$$\phi(M)dM =$$

$$= 0.4\ln(10)\phi^*\left[10^{-0.4(M-M^*)}\right]^{\alpha+1}\exp\left[-10^{-0.4(M-M^*)}\right]dM \quad (8)$$

Where ϕ^* is a normalization constant (in units of Mpc^{-3}), α is a dimensionless constant that defines the slope of the function for the dim (high magnitude) portion of the graph, and M^* is the absolute magnitude at which the Schechter function undergoes a rapid change in slope. Typical values of α are given by $-1.5 < \alpha < -1$ (Dickey 1988), although some luminosity functions have values of $\alpha > -0.1$ (Zucca et al., 2009, p. 1223). Simulated Schechter functions for various values of α are depicted in Figure 3.

In fact, one method of determining the galaxy luminosity function, the STY maximum likelihood method (Sandage, Tammann, and Yahil, 1979), simply assumes that the luminosity function has a Schechter form and then proceeds to find the values of the parameters α and M^* that provide the best fit to the data. The normalization ϕ^* may be obtained from galaxy number counts within a certain magnitude band. However, it should be noted that this normalization constant is not truly needed to obtain the selection function $S(z)$, since it is implicitly contained in both the numerator and denominator in Eq. (6) of Hebert and Lisle (2016) and thus cancels.

Solid Angles

In order to calculate a normalized luminosity function, one needs the comoving volume for the galaxy sample. This requires an appropriate solid angle Ω for the survey (but see note at the end of this section). Though physicists and mathematicians often measure solid angles in steradians (sr), the SDSS uses square degrees (see http://www.sdss3.org/dr10/), as this is more convenient for astronomers. The ratio between the two is given by:

$$4\pi \text{ steradians} \approx 41{,}253 \text{ deg}^2 \quad (9)$$

The solid angle of the survey shown in Figure 2 can be computed by integration and is also provided by the SDSS website. Once the solid angle has been obtained, the comoving volume of the survey is easily computed. The total solid angle for the tenth data release of the SDSS is cited as 14,555 square degrees $\approx$ 4.43 sr, which is a little more than 35% of the sky. However, the actual value of the solid angle is not critical, as it, like the normalization constant of the luminosity function, will "divide out" when calculating the selection function $S(z)$; see Eq. (6) in Hebert and Lisle (2016).

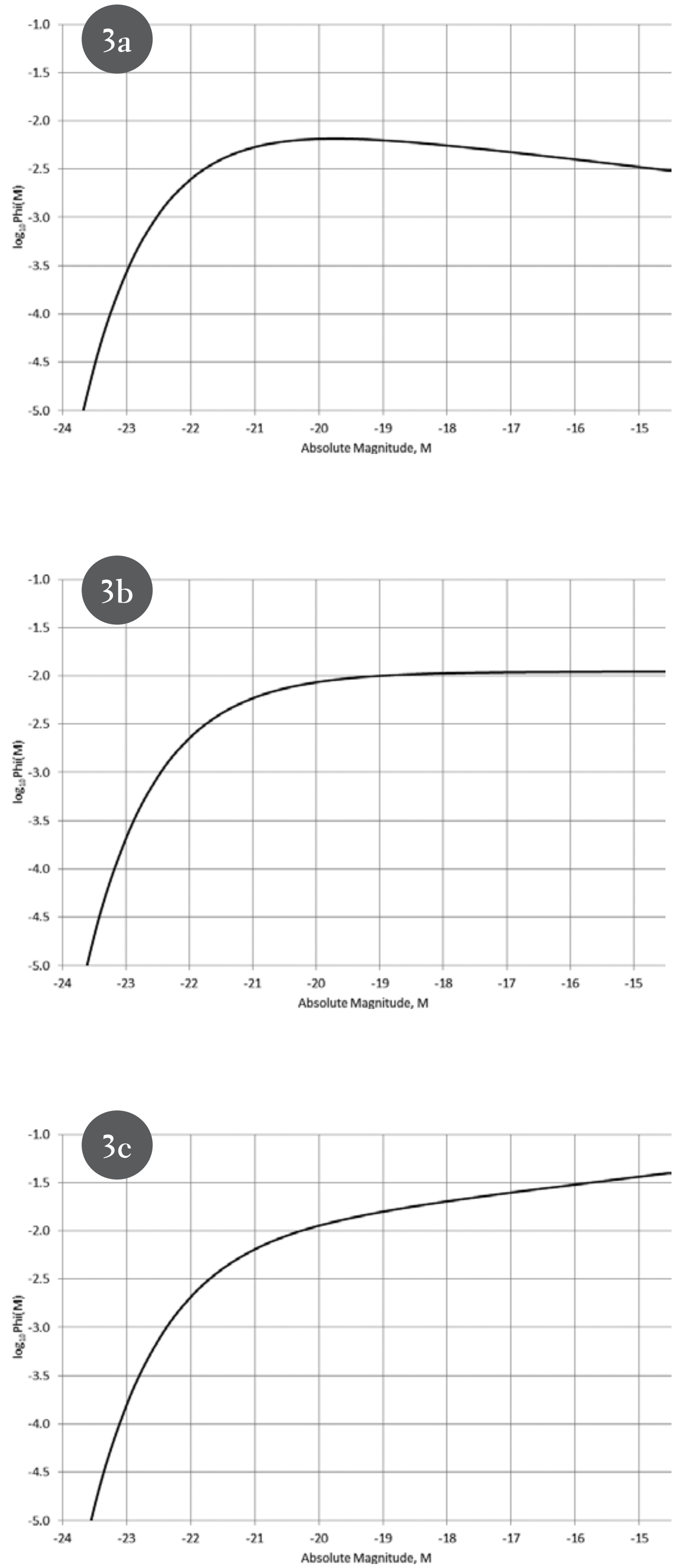

Figure 3. Simulated Schechter functions for M* = -21.5 and α equal to (a) -0.8, (b) -1.0, and (c) -1.2. Galaxy luminosity functions often have Schechter forms.

Demonstration of the Method

One of us (Lisle) has written an extremely efficient, fast, and versatile IDL code that can calculate the luminosity function for a set of galaxies via a number of different methods, including the LBC method. This demonstration uses galaxies from main Legacy survey of the tenth data release (http://skyserver.sdss3.org/dr10/en/credits/creditshome.aspx). To optimize accuracy, the code uses the *r*-band *model* apparent magnitudes when obtaining the color-dependent *K*-correction (Chilingarian, Melchior, and Zolotukhin, 2010) but *cmodel* apparent magnitudes when calculating distances. Redshift values were restricted to $0.05 \leq z \leq 0.28$ because this *K*-correction method had been tested for the *r*-band against direct *K*-corrections using galaxy flux spectra only for redshifts in this approximate range (Chilingarian, Melchior, and Zolotukhin, 2010, pp. 6–7). Galaxies having unreasonable color values (absolute value of the color greater than 3) were also excluded, as were quasars. In calculating comoving and luminosity distances, the standard values were used for cosmological parameters: $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, and $H_0 = 71.0$. A small correction was applied to redshift values to account for the motion of our solar system relative to the CMB (implicitly accepting the assumption that the CMB is not a local phenomenon and represents an average reference frame of the visible universe). Before using the LBC method, one must first *K*-correct the survey's absolute magnitudes. This means that one must make the transformations $M \rightarrow M - K$ and $\mu \rightarrow \mu + K_{avg}(z)$ for each galaxy in the survey, as discussed in Hebert and Lisle (2016) and Lisle (2016). Once this has been done, it is possible to follow Choloniewski's original method.

Next, a plot of transformed μ values versus transformed M values should be produced (Figure 4) and inspected in order to decide upon reasonable values for m_{max}, M_{min} and M_{max}.

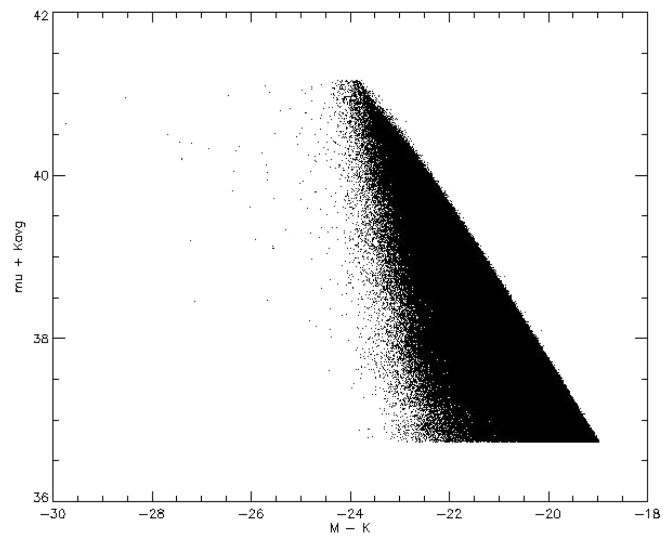

Figure 4. Plot of $\mu' = \mu + K_{avg}(z)$ versus $M_{corrected} = M - K$ for galaxies within the main survey meeting our selection criteria.

Determining m_{max}

Note that this plot differs from the basic form of Figure 1 in a number of significant ways. First, one might expect the diagonal line in Figure 4 to be very sharp and well defined, like the diagonal line in Figure 1. However, close inspection of Figure 4 reveals that this diagonal boundary is somewhat "fuzzy." This is due to the *K*-correction. The apparent magnitude cut must be applied before any *K*-correction, because it is the apparent brightness in Earth's reference frame that determines whether a galaxy can be detected. Thus, two galaxies at identical redshift and identical apparent magnitudes may have two slightly different absolute magnitudes, because they will have two different *K*-correction values. It may seem surprising that we would need to cut galaxies fainter than the limiting apparent magnitude, since these ideally should not have been included in the SDSS data set. But the SDSS cut is applied to the observed apparent magnitudes *before* any foreground contamination has been removed, whereas the final reported magnitudes have been adjusted to remove any such contamination. Thus, a galaxy significantly fainter than magnitude 17.77 could be included in the redshift survey if and only if it is very close in angle to a bright star, a nebula, or another brighter galaxy, causing the combined flux to exceed the threshold for inclusion. Since the LBC method tacitly assumes that no galaxies are detectable beyond the threshold, it has no way of dealing with these fainter galaxies. Therefore, they cannot be included in the analysis.

Because we are using cmodel rather than Petrosian fluxes to calculate galaxy apparent magnitudes, the value of m_{max} will not be exactly equal to the Petrosian cutoff of 17.77. However, we do expect it to be *close* to 17.77, due to the small differences between Petrosian and cmodel apparent magnitudes.

The optimal value of m_{max} may be estimated by preparing a histogram of cmodel *r*-band apparent magnitudes (Figure 5). From the histogram, one can see that most of the galaxies have cmodel apparent magnitudes less than ~17.8. The half-maximum value is a good choice for the brightness threshold to be used in the LBC method—a value that is clearly *less* than 17.77. An even more accurate analysis is possible that includes for observation bias (Lisle, 2016) yielding an optimal threshold of $m_r = 17.747$. Hence, for purposes of this demonstration, we set the value of m_{max} in Figures 1 and 4 to be 17.747.

Since the *K*-correction is used to estimate the absolute magnitude, both the distance estimate and apparent magnitude are unaffected; thus m_{max} is unaffected. Furthermore, since our

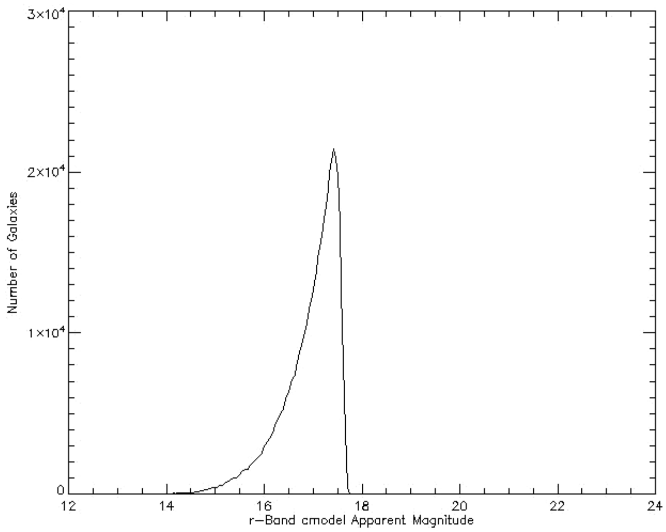

Figure 5. Histogram showing the number of galaxies in the main survey as a function of *r*-band *cmodel* apparent magnitudes. Since Petrosian apparent magnitudes are quite close in value to *cmodel* apparent magnitudes, it is not surprising that most galaxies have apparent magnitudes less than ~17.7, since the Petrosian magnitude selection cutoff was set to 17.77 (see text). Closer analysis of the data shows that the half maximum of the this sharp peak occurs at m_r = 17.747. Hence, for purposes of this demonstration of the LBC method, the value of m_{max} was set to 17.747.

transformation from μ to μ' affects only the distance coordinate, this also has no effect on m_{max}. When the limiting magnitude is plotted as a function of absolute magnitude and distance modulus, as in Figure 1, the *K*-correction (necessary to obtain the most accurate estimate of absolute magnitudes) causes the diagonal line to become a curve, due to the nonlinear response of *K* as a function of distance. Shifting to the μ' coordinate straightens this back into a diagonal line, allowing us to use the LBC method as is.

Note also that one cannot simply take m_{max} to be the very dimmest apparent magnitude of the survey. Because a significant number of galaxies have apparent magnitudes higher than this value, doing so would introduce a great deal of error into the determination of the luminosity function.

Selecting M_{min} and M_{max}

Now that m_{max} has been determined, we select appropriate values for M_{min} and M_{max}. This choice is somewhat arbitrary, though it makes sense to set these values to include most, if not all of the data. Note that it is conceivable that the survey may contain a small number of galaxies with anomalously low or high absolute magnitudes. There is no reason why these would have to be excluded from the analysis (providing they have survived the apparent magnitude cut), though we expect low statistical confidence in these extreme limits.

From Figure 4, we see that there are a small number of galaxies with absolute magnitudes less than ~ -25.0. There is no harm in setting M_{min} equal to the (*K*-corrected) absolute magnitude of the very brightest galaxy in the survey, since there is no theoretical lower limit on M_{min} and the LBC method is sufficiently robust to obtain an estimate for $\phi(M)$ even for very low (*K*-corrected) values of *M*. The solution in this magnitude range will be subject to very large errors, however, and should be considered unreliable. Likewise, M_{max} may be set to the intrinsically faintest galaxy, or to $m_{max} - \mu'_{min}$, though the paucity of extremely faint (observed) galaxies will result in low statistical confidence on this end as well.

Potential Complication: Galaxies with Equivalent Magnitudes

Choloniewski's original method assumed that no two galaxies in the survey will have exactly the same absolute magnitude. Many galaxies in the SDSS galaxy survey, however, violate this condition. Hence this complication must be considered.

One might worry that it would be necessary to completely re-derive the recursion relation between ψ_k and ψ_{k+1} found in Eq. (32) in our previous paper (Hebert and Lisle, 2016). However, this is not actually necessary. The fact that some galaxies in the SDSS survey have the same absolute magnitudes is a result of the limited precision of the survey; in reality, one does not expect any two galaxies to ever have *exactly* the same absolute magnitude. Hence, one could circumvent this difficulty by adding extremely tiny random numbers to the survey's *K*-corrected absolute magnitudes, random numbers that are much, much smaller than the precision of the recorded magnitude values. This would prevent any two galaxies in the survey from having exactly the same magnitude but without changing their values in any appreciable way. This would enable one to use the recursion relation as originally derived without modification.

However, even this is not really necessary. There are two keys to using the unmodified LBC method in this situation. The first key is to remember that the galaxies have already been sorted in order of increasing (*K*-corrected) magnitude. Hence C_k is not just equivalent to the number of galaxies inside the rectangle in our previous Figure 5; it is *also* equal to the number of galaxies that precede the k^{th} galaxy in this sorted list. This alternate definition of C_k will hold true regardless of whether or not any galaxies share the same magnitude.

The second key is to recognize that although two or more galaxies may happen to share the same magnitude, they are still

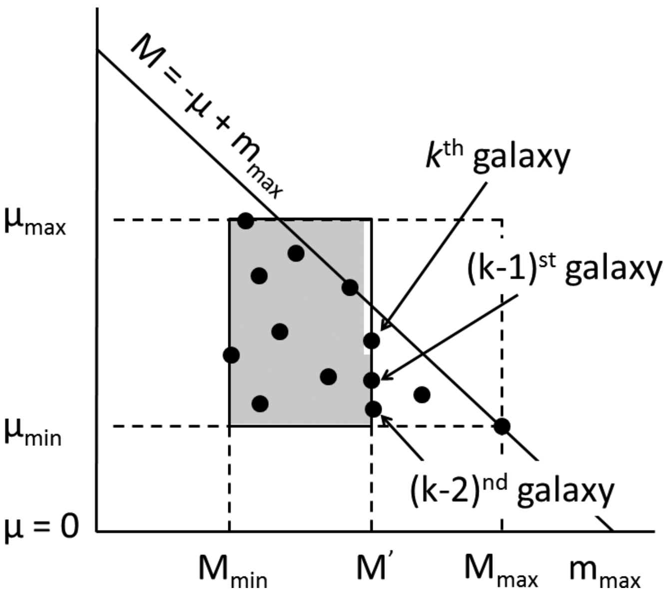

Figure 6. Diagram showing the geometry used to obtain *C (M')* when more than one galaxy has a magnitude of *M'*. Note that although we are here using Choloniewski's original notation, these are actually *K*-corrected absolute magnitudes. Likewise, the distance moduli have been transformed according to $\mu \rightarrow \mu + K_{avg}(z)$. Note also that our maximum absolute magnitude value M_{max} is very close to $m_{max} - \mu_{min}$, as one expects from Figure 4.

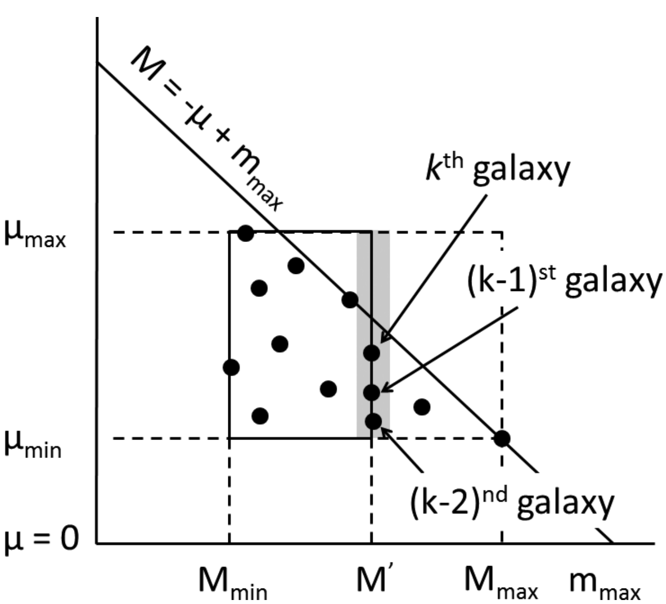

Figure 7. Geometry used to obtain Eqs. (10) and (11) in the text when multiple galaxies share the same magnitude.

characterized by *distinct* values of the index *k*. Hence, the sum on the left-hand side of Eq. (1) still contains N_{obs} terms, even though the magnitude values may not all be unique.

Now consider three galaxies (Figure 6) sharing the same magnitude M' (the reason why we are calling this magnitude M' rather than M_k as before is to prevent confusion, for reasons that should soon become apparent). We may integrate Eq. (28) in our previous paper (Hebert and Lisle, 2016) as we did before in order to obtain an expression for C_k, but this time we break the integration into two parts, the first part consists of the irregular gray region, while the second part consists of the thin, white vertical "sliver" at the upper right-hand side of our box. Integrating the left-hand side of Eq. (28) over the shaded region still gives us C_k, according to our new definition. Extending the area of integration to include the thin vertical sliver gives us one additional count due to the k^{th} galaxy. Hence, integration of the left-hand side of Eq. (28) gives us C_k+1 as before. When integrating the right-hand side of Eq. (28), we do so all at once. Because each galaxy is characterized by a unique value of *k* (even when it shares the same magnitude with another galaxy), we still get the same result as before, obtaining our previous Eq. (31). Hence, the derivation for our previous Eq. (31) is still valid, provided that we use this new, more general definition of C_k, rather than Choloniewski's old definition.

However, the derivation for our recursion relation also involved our previous Eq. (30), which contained a sum over *j*. Before we can be completely certain that our recursion relation will still hold, we must also make certain that the value of *this* sum is unaffected by galaxies with identical magnitudes. We do so by integrating this paper's Eq. (1) over the thin vertical rectangle shown in Figure 7. Integration of the left-hand side simply gives us the number of galaxies having a magnitude of M' (in this particular example, three). Integration of the right-hand side yields

$$\sum_{i:}^{M_i=M'} \psi_i \sum_{j:}^{M_i+\mu_j \leq m_{\max}} d_j$$
$$= \psi_{k-2} \sum_{j:}^{M'+\mu_j \leq m_{\max}} d_j + \psi_{k-1} \sum_{j:}^{M'+\mu_j \leq m_{\max}} d_j + \psi_k \sum_{j:}^{M'+\mu_j \leq m_{\max}} d_j$$
$$= \left(\psi_{k-2} + \psi_{k-1} + \psi_k\right) \sum_{j:}^{M'+\mu_j \leq m_{\max}} d_j \qquad (10)$$

But since all three galaxies have the same magnitude M', we expect all three values of the magnitude-dependent weighting factor ψ to be equivalent. Hence we obtain

$$3 = 3 \cdot \psi_k(M') \sum_{j:}^{M'+\mu_j \le m_{\max}} d_j \Rightarrow 1 =$$

$$= \psi_k(M') \sum_{j:}^{M'+\mu_j \le m_{\max}} d_j \quad (11)$$

which is the same as our previous result. Hence, we can continue to use our previous recursion relation without loss of generality.

One can also look at it in another way. First, imagine that we have two consecutive galaxies (numbered k and k+1 respectively) that have *almost* the same magnitude, with galaxy number k+1 being ever so slightly *fainter* than galaxy number k. And recall that $C(M_k)$ is a count of all galaxies within the rectangle associated with galaxy k but excluding galaxy k itself. Now, if galaxy k+1 is only very slightly fainter than galaxy k, then it is highly probable that $C(M_{k+1}) = C(M_k) + 1$; that is, the rectangle associated with the $(k+1)^{st}$ galaxy will have exactly one more galaxy in it (galaxy number k) than the rectangle associated with the k^{th} galaxy. This is not a certainty, because there has been a very small change in distance modulus going from k to k+1, which implies that the top of the rectangle has been reduced slightly for k+1. So it is possible that one or more galaxies at the top of the rectangle for k are excluded from the rectangle for k+1. But since the change in M (or more precisely, the change in $M_{corrected}$) was tiny going from k to k+1, the probability is low that any galaxies were lost. Moreover, that probability goes to zero in the limit as M_{k+1} approaches M_k since the change in distance modulus goes to zero:

$$\lim_{M_{k+1} \to M_k} C(M_{k+1}) = C(M_k) + 1 \quad (12)$$

Now consider two galaxies of equal magnitude. When sorted in order of decreasing magnitude, one will arbitrarily be placed in front of the other, such that one is assigned the number k and the next is assigned the number k+1. For these two galaxies $M_k = M_{k+1}$, meaning that they have the same horizontal coordinate. The key to using the LBC method with these galaxies is to use the above limit to ensure that the value of $C(M_k)$ goes up by one as k goes to k+1. In other words, when two or more galaxies share the same right edge of the rectangle, we count all galaxies within the rectangle and the edge up to but not including the k^{th} galaxy. The galaxy count $C(M)$ will therefore go up by exactly one for each consecutive galaxy of identical absolute magnitude.

Results

The LBC method was used to obtain the luminosity function via Eq. (2). The *K*-corrected absolute magnitude range was divided into 300 bins in order to obtain the result in Figure 8. Note that the luminosity function shows the least amount of noise in the middle of the magnitude range. This is expected, since galaxy counts within this magnitude region are highest. The random error increases away from the middle section, and becomes largest at either magnitude extreme since very few galaxies have these extreme magnitude values.

For the same reason, values of $\phi(M)$ for very low absolute magnitudes (<-24.5) are erratic and are not shown. This version of the LBC method does not permit a formal analysis of the errors in the luminosity function (although one could presumably use a jackknife method to do so), but another version of this method (Choloniewski 1986) does.

Potential "Pitfalls" for Future Research

A number of potential difficulties could complicate efforts to analyze galaxy patterns after compensating for the Malmquist bias using this method. First, there is the possibility that galaxies of different types may be characterized by different luminosity functions. In performing this exercise, we have used a single "global" luminosity function for all galaxies in the SDSS main

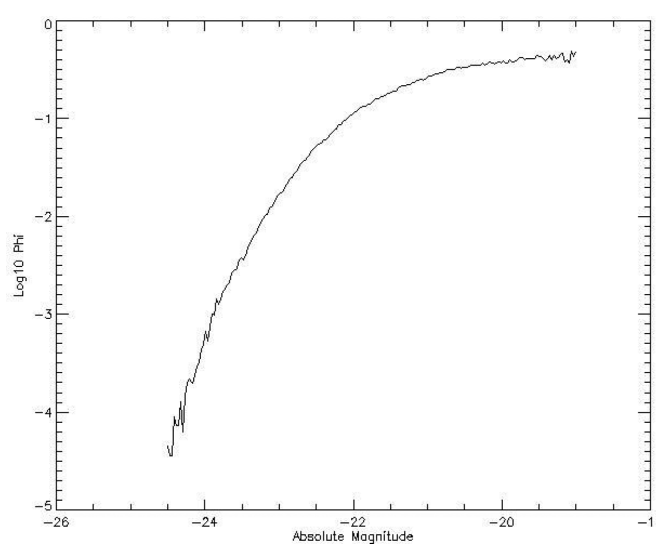

Figure 8. Luminosity function obtained using the LBC method and data from the 10th release of the Sloan Digital Sky Survey. The solution for $\phi(M)$ is not shown at values of M < -24.5, as this part of the solution is subject to large errors due to the very small numbers of galaxies within this magnitude range.

sample, regardless of galaxy type. However, Binggeli, Sandage and Tammann (1988) have noted that there are reasons to suspect that luminosity functions may vary with galaxy type. Even if this is the case, however, one would still think that it would be possible to obtain a single "global" luminosity function for all galaxies within our local vicinity. Another possibility is that the shapes of luminosity functions may vary with local density. Binggeli, Sandage, and Tammann (1988) generally reject this possibility, although they acknowledge that it might be valid for very bright galaxies.

Furthermore, most creation researchers have focused on the possibility of concentric shells of high galaxy density roughly centered on our own Milky Way galaxy, but it is also conceivable that more complicated, angular-dependent patterns might exist. These might also be indicative of design and suggestive that we occupy a special place in the cosmos.

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