A Flawed Light-in-Transit Argument (from Forty Years Ago)

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Abstract

Four decades ago, Akridge (1979) argued in *CRSQ* that energy conservation requires the in-transit creation of light from distant sources. Examination of his argument shows that, besides being flawed on *a priori* grounds, it uses an incorrect prescription for the electric field of a non-static charge; the argument is thus without merit. However, consideration of this argument provides our community with a salutary reminder of the importance of critical self-examination.

Introduction

Working through old copies of *Creation Research Society Quarterly* (in preparation for an article on the history of creationary astronomy) has been a fascinating exercise. The quantity and quality of astronomical data has increased greatly since the *Quarterly*'s inception in 1964, and the nature and sophistication of creationary argumentation has changed accordingly.

In 1979, G. Russell Akridge published a *CRSQ* article arguing for the in-transit creation of light from distant stars (Akridge, 1979). Indeed, the article's title asserts—quite boldly—that in-transit creation of light is "more than a possibility," and the article itself claims that energy conservation actually mandates this conclusion. However, subsequent issues of the *Quarterly* seem to contain neither response to nor evaluation of the argument; the lone exceptions are a brief mention by Morton (1982)—who lauds the paper as "brilliantly argued"—and another by Williams (1990)—who describes it as "a very interesting paper."

Whether brilliantly argued or not, the paper is clearly of interest because of what is at stake. If, as Akridge contends, the laws of physics demand in-transit creation of light, then the light-travel time issue simply dissolves—as does (arguably) the fundamental reality of most of astronomy. If, on the other hand, it turned out that a straightforward calculation could falsify Akridge's claim, then the article's appearance in *CRSQ* (after peer review) would be a potential embarrassment to those of us who believe in a recent creation (albeit a minor embarrassment after the lapse of forty years)—in which case, the proper recourse would be to correct the invalid claim. The occasion would also serve as a reminder of the need to exercise care in the arguments we frame, lest we inadvertently give secularists an opportunity to mock the truth. (They will indeed mock regardless—for naturalism is fundamentally presuppositional—but we should not by our own negligence hand them additional occasions to do so.)

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It is thus worthwhile, even after the passage of four decades, to consider the validity of Akridge's argument. If it is valid, then it is worth rescuing from the obscurity into which it has fallen. If, on the other hand, the argument is invalid, then it is worthwhile to use its flaws as an occasion for self-reflection. In pointing out its flaws, the goal is not to disparage a brother in Christ (who is now with the Lord) but rather to pursue the truth and to follow the biblical mandate to test all things (1 Thessalonians 5:21).

Thus, in the following sections we first summarize Akridge's argument and present some preliminary considerations. We then perform a basic electrodynamic calculation to evaluate the argument's merit, and we close with some reflections on the importance of self-evaluation to our community. This article assumes some familiarity with classical electromagnetic theory; Appendix A provides a brief review of Maxwell's Equations and the Dirac delta function. In addition, this article does not seek to address (or take a position on) in-transit creation of light *per se*; its focus is specifically Akridge's argument and its implications.

Overview of the Argument

Akridge's argument is easy to state: after the initial creation of a charge, its electric field **E** must propagate outward at the speed of light. Thus the field will occupy an increasing volume of space with the passage of time. However, the energy U of an electromagnetic field depends on $E^2 + B^2$, integrated over all space (B being the magnetic field strength, which in this case vanishes). Using the standard expression for the electric field of a static point charge, Akridge argues that as the field expands and occupies more volume, its total energy will increase as well; therefore he claims that the expanding field violates energy conservation.

Hence, Akridge concludes that the electric fields of all charged particles must have been created at the same instant as the particles themselves; furthermore, these fields (rather than expanding) must at the moment of their creation have extended throughout all of space. Thus, since light consists of electric and magnetic fields, the light of all luminous bodies must have been created (throughout all of space) at the instant the bodies themselves were created. Therefore, he claims, the light from distant stars was created in transit, and the long ages inferred from distant starlight are not problematic to a young-earth position.

Evaluation

A priori considerations

It should be clear at the outset—before performing any calculations—that something is wrong with this argument. For instance, pair production (of an electron-positron pair from a photon, $\gamma \rightarrow e^- + e^+$) is a common occurrence in high-energy physics; it regularly creates positive and negative charges *de novo* (while conserving net charge). And though the fields of the new particles will almost completely cancel out (their charges being equal and opposite), the cancellation will not be perfect at any time after their creation, simply because the particles do not occupy the same positions in space. Hence, if Akridge's argument were sound, these newly created fields would simultaneously spring into existence throughout all of space—whereas in reality, the fields propagate with the speed of light.

In addition, if energy conservation requires simultaneous creation of electric fields throughout space, then energy conservation is inconsistent with special relativity. For, by relativity of simultaneity, the field creation can be simultaneous in only one set of inertial frames, and thus (were Akridge correct) energy could be conserved in those frames only. But in fact Maxwell's equations provide a Lorentz-invariant theory which guarantees conservation of energy in all inertial frames.

Furthermore, the argument makes some significant assumptions. For instance, even if we grant the simultaneous creation of these fields, on what grounds would we expect them to oscillate, as they must do in order to constitute light? Why would the fields not remain static until the motion of the charges disturbs them? Or, if the fields are not static, why would they not be randomly fluctuating—given that they are causally disconnected from the charges that (somehow) are responsible for generating them? And even if (somehow) the fields were to oscillate and propagate as light, why would this light encode an entire coherent virtual history?

These problems signal the existence of a fundamental flaw in the argument itself; we now turn to an examination of that flaw.

Calculating the electric field

Akridge's error lies in his equation for the electric field of a non-static charge. He begins with the unobjectionable fact that the strength of the electric field r units distant from a static charge q is $E(r)=q/4\pi\epsilon_0 r^2$ (where ϵ_0 is the permittivity of free space). Akridge then assumes that the field of a newly created charge is exactly like the field of a static charge except for being truncated (after time t) at a radius ct (where c is the speed of light). In other words, he assumes that the field strength at time t after the creation of the charge q is

$$E(r,t) = \begin{cases} \dfrac{q}{4\pi\epsilon_0 r^2} & \text{if } r < ct \\ 0 & \text{if } r > ct. \end{cases} \qquad (1)$$

Equation 1 is the prescription employed by Akridge (in the fourth paragraph of his article) for the electric field of a newly created charged particle. If this prescription is correct, then the remainder of his argument follows.

However, reference to standard electrodynamics texts quickly reveals that Equation 1 is *not*, in fact, the correct prescription for the electric field of a varying charge. Although the electric potential V does behave in such a manner, the electric field **E** does not. (See, for instance, Griffiths, 1999, pp. 423–424, or other standard electrodynamics texts.) Ironically, Akridge's own Appendix A demonstrates that the expression in Equation 1 describes not the creation of a simple point charge but rather the outward propagation of a shell of charge. He does not seem to notice the contradiction between this result and the rest of his paper.

To calculate the true field of a varying point charge, we should instead begin with a time-dependent charge density $\rho(\mathbf{r},t)$; we can then calculate the resulting time-dependent potential $V(\mathbf{r},t)$, the gradient of which yields the electric field $\mathbf{E}(\mathbf{r},t)$. Alternatively, one could use Jefimenko's equations (see, e.g., Griffiths, 1999), which directly link the electric and magnetic fields to the charge configuration $\rho(\mathbf{r},t)$ and current density $\mathbf{J}(\mathbf{r},t)$. It is this approach which we here take. Jefimenko's equation for the electric field is

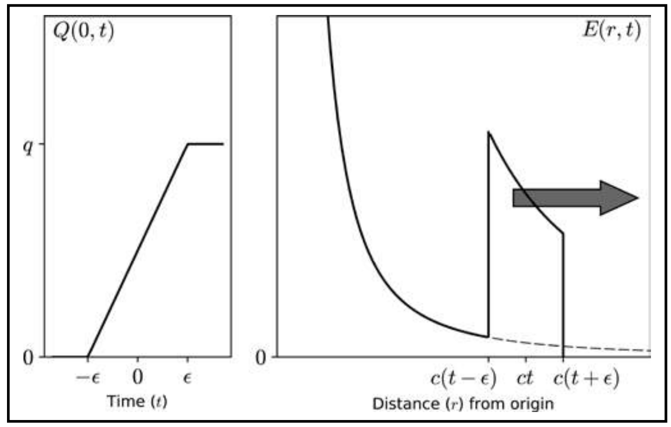

Figure 1. Left panel: The rise of a point charge at the origin (Equation 4); to avoid discontinuities, we assume a linear rise from 0 to q over a time period of 2ϵ. Right panel: The magnitude E (Equation 5) of the electric field at a given time t, as a function of distance from the origin. The dashed line shows the electric field of a static charge; as time increases, the "spike" about $r=ct$ propagates to the right and diminishes in size so that the field approaches the static limit. Compare this—the true field configuration—with Equation 1, which is assumed by Akridge (1979) and lacks the spike.

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int\left(\frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}') + \frac{\dot{\rho}(\mathbf{r}',t_r)}{c|\mathbf{r}-\mathbf{r}'|^2}(\mathbf{r}-\mathbf{r}') - \frac{\mathbf{J}(\mathbf{r}',t_r)}{c^2|\mathbf{r}-\mathbf{r}'|}\right)d^3r', \tag{2}$$

where the retarded time $t_r=t-|\mathbf{r}-\mathbf{r}'|/c$ enforces the finite propagation speed of disturbances in the field. The corresponding equation for the magnetic field is

$$\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\left(\frac{\mathbf{J}(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}',t_r)}{c|\mathbf{r}-\mathbf{r}'|}\right) \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}d^3r', \tag{3}$$

where μ_0 is the permeability of free space.

So let us begin by modeling the creation of a point charge q at the origin. We shall work in the rest-frame of the charge so that the current density **J**—and thus, by Equation 3, the magnetic field—vanishes. (See below for additional discussion of currents and magnetic fields in this situation.) We shall also avoid unnecessary discontinuities by positing that the creation of the charge occurs, in a linear fashion, over the time span $-\epsilon<t<\epsilon$; later, if we wish, we can consider the limit as ϵ approaches zero. Thus, the time-dependent charge distribution giving rise to the electric field is (see Figure 1)

$$\rho(\mathbf{r},t) = \begin{cases} 0 & \text{if } t \leq -\epsilon \\ \frac{q}{2}\left(\frac{t}{\epsilon}+1\right)\delta_D(\mathbf{r}) & \text{if } -\epsilon < t < \epsilon \\ q \cdot \delta_D(\mathbf{r}) & \text{if } t \geq \epsilon, \end{cases} \tag{4}$$

where $\delta_D(\mathbf{r})$ is the three-dimensional Dirac delta function (see Appendix A). Next, application of Equation 2 to the distribution in Equation 4 yields (see Appendix B for details) the following electric field:

$$\mathbf{E}(\mathbf{r},t) = \begin{cases} \dfrac{q \cdot \hat{\mathbf{r}}}{4\pi\epsilon_0 r^2} & \text{if } r \le c(t-\epsilon) \\ \dfrac{q \cdot \hat{\mathbf{r}}}{8\pi\epsilon_0 r^2} \cdot \dfrac{t+\epsilon}{\epsilon} & \text{if } c(t-\epsilon) < r < c(t+\epsilon) \\ 0 & \text{if } r \ge c(t+\epsilon). \end{cases} \tag{5}$$

If we now compare Equation 5 to the (incorrect) prescription of Equation 1, we see that the essential difference is a "spike" in the field (see Figure 1) produced during the charge creation. It is this spike (from the $\dot{\rho}$ term in Equation 2, and absent in Akridge's analysis) which maintains conservation of energy once the charge has been created.

We can explicitly verify energy conservation by calculating the total electromagnetic energy

$$U(t) = \frac{1}{2}\int \left(\epsilon_0 E(\mathbf{r},t)^2 + \frac{1}{\mu_0} B(\mathbf{r},t)^2\right) d^3r \tag{6}$$

at all times $t \geq \epsilon$. In this frame there is no magnetic field **B**, and thus we need integrate only over E^2; the details of the calculation appear in Appendix B, and the result is a constant energy

$$U(t) = \frac{q^2}{(2\epsilon)8\pi\epsilon_0 c} + \frac{q^2}{8\pi\epsilon_0 r_0}, \tag{7}$$

where r_0 is the lower limit of integration. (A positive lower limit is necessary to avoid divergence, just as in the standard calculation of the energy of a point charge.)

It is evident that time does not appear in Equation 7; hence, after the charge has been created (i.e., for $t \geq \epsilon$), the energy of its field is constant. Thus the expanding electric field does not violate energy conservation, and therefore instantaneous field-creation throughout the universe is not required. Akridge's argument is hence without merit.

Analysis

The second of the two terms in Equation 7 is no surprise—it is the standard expression for the energy of a static charge. The first term, then, is in some way the result of charge creation over a time span of length 2ϵ. The more gradual the charge creation, the less significant the term becomes, whereas it becomes infinite in the limit $\epsilon \to 0$. It is thus worthwhile to consider this term more closely.

One feature of Maxwell's equations is that they impose local charge conservation automatically; the standard derivation of this result begins with Maxwell's modification of Ampere's Law (Equation A.5) and takes the divergence:

$$\nabla \times \mathbf{B} - \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \tag{8}$$

$$\nabla \cdot (\nabla \times \mathbf{B}) - \mu_0\epsilon_0 \frac{\partial}{\partial t} \nabla \cdot \mathbf{E} = \mu_0 \nabla \cdot \mathbf{J} \tag{9}$$

Since the divergence of a curl vanishes, we conclude from Gauss's Law (Equation A.2) that

$$-\frac{\partial}{\partial t}\rho = \nabla \cdot \mathbf{J}; \tag{10}$$

that is, an increase (or decrease) of charge at any point must be accomplished by a flow of current into (or out of) that point. This statement clearly contradicts our postulated scenario, in which there exists a positive $(\partial/\partial t)\rho$ with no corresponding current **J**. Thus, we (along with Akridge) are actually applying Maxwell's equations to a situation in which they manifestly do not hold; it is therefore no wonder that we obtain an unexpected term in Equation 7.

Consideration of the magnetic field produces the same conclusion. By symmetry considerations, the field **B** must point in a radial direction; however, the vector field $d\mathbf{E}/dt$ is also strictly radial, and it must be orthogonal to **B** (by Equation A.5). Thus **B** must vanish, as we have already concluded on the basis of Equation 3. On the other hand, the Maxwell-Ampere law (Equation A.5) requires that a growing charge (with no magnetic field) *must* be accompanied by a current—and yet the only charge in the Universe (in our toy model) is at rest, meaning that there can be no current.[1] Indeed, this contradiction simply restates the continuity issue of the preceding paragraph, and the conclusion is the same—both we and Akridge have been applying Maxwell's equations to a situation in which they do not hold.

We could instead have "created" (or rather, accumulated) the charge by means of a current flowing in from infinity: for

1 It is of course also true that charged elementary fermions possess an intrinsic magnetic moment due to their spin. Inclusion of this quantum mechanical effect would be an interesting study but would not alter the conclusions of this paragraph.

instance, one could postulate (to maintain spherical symmetry) that $\mathbf{J}(\mathbf{r},t)=-q\hat{\mathbf{r}}/8\pi\epsilon r^2$ during the time $-\epsilon<t<\epsilon$, so that $\nabla\cdot\mathbf{J}$ gives us $\dot{\rho}(\mathbf{r},t)$ matching Equation 4 (for all times t and points $\mathbf{r}$). For this scenario, the reader can verify (after rather tedious integration) that the anomalous term in Equation 7 disappears, although Equation 5 changes significantly due to the simultaneous currents filling all of space. Interestingly, in this case also the magnetic field turns out to vanish, with the $\mathbf{J}$ and $d\mathbf{E}/dt$ terms in Equation A.5 cancelling out—and thus we remain consistent with Maxwell's Equations.

If on the other hand one wishes to model charge creation *per se* (without recourse to any such currents), then there are at least two options. First, it is possible that when God spoke charge into existence, He chose not to maintain local charge conservation. Indeed, since God ceased His creative work after Day 6, we cannot assume that today's conservation laws (describing His ongoing work of providence) necessarily apply to His work of creation. In such a case, modeling the event would require a self-consistent formulation of electrodynamics without local charge conservation. It is unclear what form such a theory would take: on the quantum level, one could perhaps introduce appropriate operators for creation/annihilation of charge, while taking care to account for gauge invariance, particle spin, etc.; on the classical level, one could perhaps adapt Maxwell's equations to a non-trivial topology and model the introduction of charge through some sort of wormhole. In either case, the work required is well beyond the expertise and inclination of the author. Nor would the utility of such a theory (or its amenability to empirical test) be at all clear.

The second option (perhaps preferable on aesthetic grounds) is that when God first spoke charge into existence, He did so using a process similar to that by which He upholds the universe today—namely, by always producing equally- and oppositely-charged pairs of particles. Two examples are pair production ($\gamma\rightarrow e^-+e^+$) and beta decay ($n\rightarrow e^-+p^++\overline{\nu}_e$). Since the Universe is (to our knowledge) electrically neutral, it is probable that *global* charge conservation has existed from the beginning, and thus it is not unreasonable to suppose that local charge conservation has held true as well.

Neither option, however, requires in-transit light-creation. If God utilized some sort of pair production consistent with Maxwell's Laws, then those laws insure both energy conservation and the propagation of field disturbances at the speed of light. If God utilized some other sort of method, then He would have been free either to create light in transit or not.

Conclusions

Akridge (1979) argues that conservation of energy requires instantaneous creation of universe-wide electric fields whenever electric charge is created; he thus concludes that light from distant stars was created in transit. We have demonstrated that this argument is fundamentally flawed, on both *a priori* and electrodynamical grounds. In particular, Akridge employs an incorrect prescription for the time-dependent electric field; in contrast, the correct prescription satisfies energy conservation automatically.

However, we should note that this argument, with its flaws, provides our community an opportunity for self-reflection. Given the intervening four decades and the relative obscurity of the argument, we have the rare advantage of being able to perform this self-reflection with minimal risk of personal offense. What conclusions, then, can we draw from this incident in the history of creationism?

We first note that Akridge's argument seems never to have gained much traction within creationary circles. Indeed, apart from the two mentions noted above, the argument seems to have languished in relative obscurity. If this fact indicates skepticism among the CRS membership concerning the argument's merits, then we have demonstrated that the unease was well-warranted. It seems likely that many members sensed its problematic nature, and thus our history gives us a positive example of caution in handling arguments that do not "sound" plausible, even if we are unable immediately to pinpoint the precise nature of that implausibility.

However, we must also note that the argument did pass peer review and appeared in *CRSQ*, arguably the flagship journal of creation science. On the one hand, we should make full allowance for the acute difficulty of finding willing and qualified reviewers during the early decades of the Creation Research Society. On the other hand, it seems unlikely that the CRS at that time included absolutely *no* members conversant enough with electrodynamics to pinpoint these flaws. In any case, and for whatever reason, neither the reviewer(s) nor anyone within the 1979 membership seems to have pointed them out.

To take note of this failure is not to suggest that individual creationists are somehow responsible for other creationists' errors, much less errors decades in the past. Nor is today's organization to be blamed for a forty-year old misstep. Nevertheless, the error under discussion is by no means minor: the argument proceeds from an unambiguously false electrodynamic premise, and peer review is intended to detect precisely this type of error. Thus, remembering the aphorism about those who forget history, it is worthwhile to consider—not what went wrong in 1979, but rather how we could prevent (or at least quickly redress) a similar error today.

The answer must ultimately involve our organizational responsibility to self-regulate, a responsibility which inheres in no one member exclusively but in the community as a whole. What then falls to each of us (as members of the community) is to thoughtfully consider, evaluate, and interact with the articles and arguments which lie within our own individual areas of expertise. Sometimes doing so is straightforward. At other times, it can be difficult to critique ideas which one feels *ought* to be true. Nevertheless, "the first to present his case seems right until another comes forward and questions him" (Proverbs 17:19)—and we, rather than unbelievers, should be the first to raise such questions in a rigorous and yet constructive fashion.

Indeed, this is precisely what CRS *has* been doing for over five decades, and during that time there has been significant refinement in the quality (and scope) of creationary argumentation in virtually all areas. We can be grateful for this growth in the caliber of the science published in *CRSQ*, and we can be even more grateful that, in the process, the organization has remained faithful to Scripture. Both of these blessings are the fruit of the selfless work of many CRS members and, especially, of the grace of God. But repeated reminders—of the importance of excellent science, accurate exegesis, and doctrinal fidelity—are wholesome, and the author hopes that this examination of Akridge (1979) can serve as another such reminder. Our faith in God's Word will always incur ridicule from the unbelieving world, but we surely do not wish to invite such ridicule by our own failure to engage in critical thinking. In addition, we have a much greater motive for accurate argumentation, namely, the account we must each give to our Creator and Redeemer: "we make it our aim to please Him, for we must all appear before the judgment seat of Christ" (2 Corinthians 5:9–10), and "each one's work will become manifest … what sort of work each one has done" (1 Corinthians 3:13).

Thus it is incumbent upon us as a community—authors, readers, and reviewers (each within his own area of expertise)—to subject our ideas to searching critique by carefully examining their assumptions and implications. To borrow biblical language, the more diligently we judge ourselves, the less likely it is that we shall be judged wanting. Thus, the flaws in this argument from a previous generation can serve as a wholesome stimulus for us today (including this author) to thoughtfully weigh the positions we take in defense of the biblical record—and to critique our own arguments as carefully as we critique others'.

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Appendix A
Maxwell's Equations and the Dirac Delta

This appendix provides a brief review of Maxwell's equations and the Dirac delta function, employing SI units throughout. For a more comprehensive explanation, one can consult any standard electrodynamics text; the exposition of Griffiths (1999) is particularly lucid.

Electric and magnetic fields exert force on electric charges. The Lorentz force law describes this effect by specifying the force exerted on a small test charge q with velocity $\mathbf{v}$ by electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, respectively:

$$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B}). \qquad \text{(A.1)}$$

Note that these fields, like the resulting force, are vector quantities (in the mathematical sense).

Whereas the Lorentz force law describes the effect of fields on charges, Maxwell's Equations describe the fields generated by the charges. Specifically, given a charge density ρ and a current density $\mathbf{J}$, one can write Maxwell's Equations as follows:

$$\nabla\cdot\mathbf{E}=\frac{1}{\epsilon_0}\rho \qquad \text{(A.2)}$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t} \qquad \text{(A.3)}$$

$$\nabla\cdot\mathbf{B}=0 \qquad \text{(A.4)}$$

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}+\mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}. \qquad \text{(A.5)}$$

In these equations, ϵ_0 (the permittivity of free space) is $8.85\times10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$, and (the permeability of free space) is $4\pi\times10^{-7}\ \mathrm{N/A^2}$. The four equations A.2–A.5, together with the Lorentz force law (A.1), constitute the foundation of classical electromagnetic theory. From these equations, one can derive the following expression for the energy U of an electromagnetic field:

$$U(t)=\frac{1}{2}\int\left(\epsilon_0E(\mathbf{r},t)^2+\frac{1}{\mu_0}B(\mathbf{r},t)^2\right)d^3r. \qquad \text{(A.6)}$$

Next, the so-called Dirac delta function facilitates the application of Equations A.2–A.5 to point charges. The Dirac delta is not, strictly speaking, a function but rather a distribution (or, alternatively, a measure). One can visualize it as an infinitely narrow and infinitely tall spike at the origin. Thus one can write (suggestively)

$$\delta_D(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0. \end{cases} \tag{A.7}$$

so that (precisely)

$$\int_{-\infty}^{\infty} \delta_D(x)\, dx = 1. \tag{A.8}$$

Note that Equation A.7 requires Equation A.8 (or, better, Equation A.9 below) to render its meaning precise. One can also consider the Dirac delta to be the limit of Gaussian probability distributions with mean 0 and standard deviation approaching zero (as in Figure 2).

The key property of a delta function is its ability to pick out one value from an integrand, so that for any function *f(x)* we have

$$\int_{-\infty}^{\infty} f(x)\delta_D(x-c)\, dx = f(c). \tag{A.9}$$

The three-dimensional Dirac delta function is simply the product of one-dimensional deltas:

$$\delta_D(\mathbf{r}) = \delta_D(x)\cdot\delta_D(y)\cdot\delta_D(z), \tag{A.10}$$

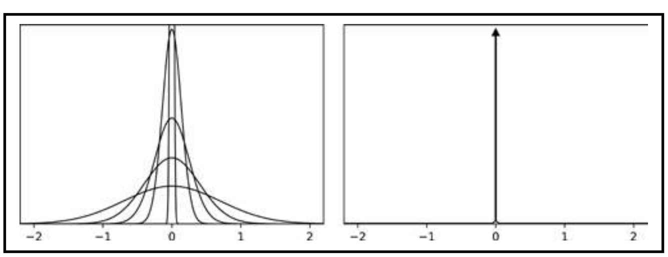

Figure 2. The left-hand panel shows a series of Gaussian functions (of unit area) converging to the Dirac delta, depicted in the right-hand panel. The Dirac delta "function" is zero everywhere except at the origin, where its value is infinite.

so that

$$\int f(\mathbf{r})\delta_D(\mathbf{r}-\mathbf{r}_0)\, d^3r = f(\mathbf{r}_0). \tag{A.11}$$

Appendix B
Deriving Equations 5 and 7

This appendix provides the derivation of Equation 5 from the charge distribution of Equation 4; it then provides the derivation of Equation 7 from Equation 5.

Recall that the electric field for any configuration of charges and currents is given by the first of Jefimenko's equations (Equation 2), which we reproduce here:

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int\left(\frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}') + \frac{\dot{\rho}(\mathbf{r}',t_r)}{c|\mathbf{r}-\mathbf{r}'|^2}(\mathbf{r}-\mathbf{r}') - \frac{\mathbf{J}(\mathbf{r}',t_r)}{c^2|\mathbf{r}-\mathbf{r}'|}\right)d^3r', \tag{B.1}$$

In our scenario, the current density **J** vanishes, and Equation 4 gives the charge density:

$$\rho(\mathbf{r},t) = \begin{cases} 0 & \text{if } t \leq -\epsilon \\ \frac{q}{2}\left(\frac{t}{\epsilon}+1\right)\delta_D(\mathbf{r}) & \text{if } -\epsilon < t < \epsilon \\ q\cdot\delta_D(\mathbf{r}) & \text{if } t \geq \epsilon, \end{cases} \tag{B.2}$$

Taking the time derivative, we obtain

$$\dot{\rho}(\mathbf{r},t) = \begin{cases} \frac{q}{2\epsilon}\delta_D(\mathbf{r}) & \text{if } -\epsilon < t < \epsilon \\ 0 & \text{otherwise.} \end{cases} \tag{B.3}$$

We now must calculate two integrals, one involving ρ and the other involving $\dot{\rho}$, with both integrands evaluated at retarded time $t_r=t-|\mathbf{r}-\mathbf{r}'|/c$. The Dirac delta function makes the integration straightforward and enforces the condition $\mathbf{r}'=0$, so that $t_r=t-r/c$. Thus we have

$$\int d^3r' \frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}') = C\cdot\frac{\hat{\mathbf{r}}}{r^2},$$

$$\text{where } C = \begin{cases} 0 & \text{if } r \geq c(t+\epsilon) \\ \frac{q}{2}\left(\frac{1}{\epsilon}\left(t-\frac{r}{c}\right)+1\right) & \text{if } c(t-\epsilon) < r < c(t+\epsilon) \\ q & \text{if } r \leq c(t-\epsilon), \end{cases}$$

(B.4)

and

$$\int d^3r' \frac{\dot{\rho}(\mathbf{r}',t_r)}{c|\mathbf{r}-\mathbf{r}'|^2}(\mathbf{r}-\mathbf{r}') = D\cdot\frac{\hat{\mathbf{r}}}{cr},$$

$$\text{where } D = \begin{cases} \frac{q}{2\epsilon} & \text{if } c(t-\epsilon) < r < c(t+\epsilon) \\ 0 & \text{otherwise.} \end{cases}$$

(B.5)

With these integrals in hand, we can evaluate Equation B.1 as a function of distance at any particular time t. First, at distances $r \geq c(t+\epsilon)$, there has not yet been enough time for the effect of the charge at the origin to propagate out to r, and we have

$$\mathbf{E}(\mathbf{r},t) = 0 \text{ for } r \geq c(t+\epsilon). \tag{B.6}$$

Next, if $c(t-\epsilon)<r<c(t+\epsilon)$, then the electric field at point $\mathbf{r}$ feels the effect of the growing charge and is in the "spike" portion of Figure 1:

$$\mathbf{E}(\mathbf{r},t) = \frac{\hat{\mathbf{r}}q}{4\pi\epsilon_0}\left(\frac{1}{2r^2}\left(\frac{1}{\epsilon}\left(t-\frac{r}{c}\right)+1\right)+\frac{1}{2\epsilon cr}\right)$$

(B.7)

$$= \frac{\hat{\mathbf{r}}q}{8\pi\epsilon_0 r^2}\cdot\frac{t+\epsilon}{\epsilon}, \text{ for } c(t-\epsilon) < r < c(t+\epsilon).$$

(B.8)

Finally, if $r \leq c(t-\epsilon)$, the field is that expected for a static charge:

$$\mathbf{E}(\mathbf{r},t) = \frac{\hat{\mathbf{r}}}{4\pi\epsilon_0}\frac{q}{r^2} \text{ if } r \leq c(t-\epsilon).$$

(B.9)

Together, Equations B.6, B.8, and B.9 yield Equation 5:

$$\mathbf{E}(\mathbf{r},t) = \begin{cases} \frac{q\cdot\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2} & \text{if } r \leq c(t-\epsilon) \\ \frac{q\cdot\hat{\mathbf{r}}}{8\pi\epsilon_0 r^2}\cdot\frac{t+\epsilon}{\epsilon} & \text{if } c(t-\epsilon) < r < c(t+\epsilon) \\ 0 & \text{if } r \geq c(t+\epsilon). \end{cases}$$

(B.10)

This expression for $\mathbf{E}(\mathbf{r},t)$—along with the fact that the magnetic field vanishes by symmetry—allows us to calculate the net electromagnetic energy at any time t via Equation 6:

$$U(t) = \frac{1}{2}\int\left(\epsilon_0 E(\mathbf{r},t)^2 + \frac{1}{\mu_0}B(\mathbf{r},t)^2\right)d^3r$$

(B.11)

Proceeding, we have

$$U(t) = \frac{1}{2}\int \epsilon_0 E(\mathbf{r},t)^2\, d^3r = 2\pi\epsilon_0\int E(r,t)^2\, r^2 dr$$

(B.12)

$$= 2\pi\epsilon_0 q^2 \left\{ \frac{(t+\epsilon)^2}{64\pi^2\epsilon_0^2\epsilon^2} \int_{c(t-\epsilon)}^{c(t+\epsilon)} \frac{dr}{r^2} \right.$$

$$\left. + \frac{1}{16\pi^2\epsilon_0^2} \int_{r_0}^{c(t-\epsilon)} \frac{dr}{r^2} \right\} \quad \text{(B.13)}$$

where r_0 is an arbitrary (positive) lower limit required for convergence of the integral. Continuing,

$$U(t) = \frac{q^2(t+\epsilon)^2}{32\pi\epsilon_0\epsilon^2} \left(\frac{1}{c(t-\epsilon)} - \frac{1}{c(t+\epsilon)} \right)$$

$$+ \frac{q^2}{8\pi\epsilon_0} \left(\frac{1}{r_0} - \frac{1}{c(t-\epsilon)} \right) \quad \text{(B.14)}$$

$$= \frac{q^2(t+\epsilon)}{16\pi\epsilon_0\epsilon c(t-\epsilon)} + \frac{q^2(c(t-\epsilon)-r_0)}{8\pi\epsilon_0 r_0 c(t-\epsilon)} \quad \text{(B.15)}$$

$$= q^2 \frac{r_0 + 2\epsilon c}{16\pi\epsilon_0\epsilon c r_0} \quad \text{(B.16)}$$

$$= \frac{q^2}{16\pi\epsilon_0\epsilon c} + \frac{q^2}{8\pi\epsilon_0 r_0}, \quad \text{(B.17)}$$

as in Equation 7.