Using Vardiman's Young-Earth Ice Sheet Model and a Simple Computer Code to Estimate Annual Layer Thicknesses

Jake Hebert*

Abstract

Unfortunately, uniformitarian models of the Greenland and Antarctic ice sheets are currently much more sophisticated than creation models. In fact, creation researchers are about sixty years behind uniformitarian scientists in this area. There is a great need for creation researchers to develop more sophisticated ice sheet and ice core models. Creation Ice Age models predict that annual layers at given depths in the deep Greenland and Antarctic ice cores will be considerably thicker than predicted by uniformitarian models. Here I use Larry Vardiman's 1993 analytical ice sheet model and a simple computer code to obtain estimates of annual layer thicknesses in Greenland's Camp Century ice core. These results are admittedly of limited usefulness, but it is my hope that this paper will help lead to better creation-based ice sheet models. I also hope that it serves a pedagogical purpose in enabling future creation researchers to quickly grasp important ice sheet concepts and to avoid misconceptions.

Introduction

In 1993, Larry Vardiman of the Institute for Creation Research published an analytical model for the rapid post-Flood formation of a thick ice sheet within the 4,500 years since the Genesis Flood (Vardiman, 1993, 1994, and 2001). Other creation researchers (Sherburn, Horsteymeyer, and Solanki, 2008) have used commercial software to model surging (short-term rapid advances) of glaciers. However, very little additional work has been done in this field by creationists, and creation research in this area lags secular research by about sixty years. In particular, there is a need for more sophisticated models for the rapid growth of post-Flood ice sheets, as well as a need to make predictions of annual layer thicknesses, to enable contrasts with the predictions of secular ice sheet models. This paper is a modest first step in that direction. Because of the much smaller number of available creation researchers, it is

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very important to clearly explain important concepts to future creation researchers in this field, so as to minimize the time needed for them to get "up to speed." Writing this paper has been a useful exercise that has helped clarify my thinking on a number of issues, and I hope that it may serve as a tutorial or primer that enables future creation researchers to quickly grasp ice sheet basics and to avoid misconceptions.

Ice Sheet Basics

Glaciologists often drill ice cores at geological features called *ice divides*. These features derive their name from the fact that ice on one side of the divide flows one way, and ice on the other side of the divide flows the other (Figure 1). Ice is deposited on the sheet in horizontal layers. Ice at the divide itself will flow neither to the right nor left, but will move straight down as additional layers are added, assuming no shifting of the ice divide during ice buildup. It is also possible for the ice to form a dome around the divide, so that ice flows radially outward from the divide, not just right and left.

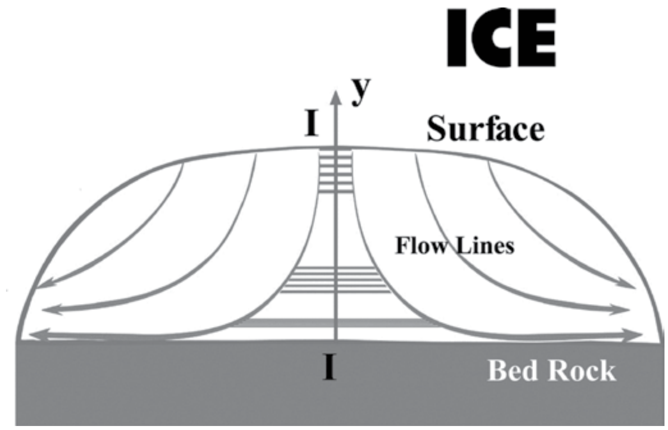

Figure 1. An idealized ice divide. Note that ice on the left of the divide flows to the left, and ice on the right of the divide flows to the right. Ice at the divide location must move straight down. Image courtesy of Michael J. Oard.

Since the underlying bedrock is fixed (here we ignore possible isostatic adjustments), a horizontal layer that reaches bedrock can no longer move downward; it can only become thinner with time. If no melting occurs at bedrock, the total number of annual layers will equal the number of years since the ice sheet began forming. However, identifying the true number of annual layers is not trivial, since multiple distinct layers may be deposited per year (Alley and Koci, 1988; Alley, 1988). In fact, creation critic Bill Nye inadvertently highlighted why one *cannot* naively assume that each visible band in an ice core is an annual layer (Hebert, 2018a). The counting process has been described as an "art" that requires "trial and error" (Alley et al., 1997, pp. 26367, 26370). Hence uniformitarian glaciologists must make educated guesses about how many of these distinct bands should be grouped together and counted as a year. Unfortunately, these educated guesses are biased by long-age assumptions and circular reasoning (Alley et al., 1997, p. 26419; Oard, 2005; Hebert, 2014).

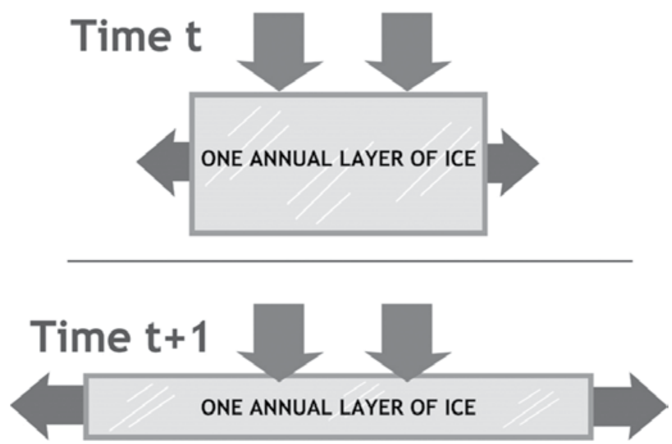

Figure 2. As more layers of ice are deposited at the site of the divide, the layers of ice become thinner, but in such a way that the total volume of each layer remains constant. Image courtesy of Michael J. Oard.

It should be mentioned in passing that it may technically be a misnomer to say that these ice layers are "compressed." Simple ice sheet models often ignore the variations in density due to the presence of bubbles in the uppermost parts of the ice (which is actually a combination of ice and granular snow called *firn*). The density of glacial ice is 917 kg/m^3, and temperature and pressure variations at the bottom of a four-kilometer-thick ice sheet will cause only slight increases in density (Cuffey and Paterson, 2010, pp. 12–13) to about 921 to 922 kg/m^3, a less than 1% difference. Hence, at most depths, the ice density will be very nearly constant, and the assumption of incompressibility (constant density and volume) is usually made (Paterson, 1980, p. 12).

Hence, in the absence of melting or ablation, the total volume of an ice layer remains constant over time. The layer becomes thinner, but it does so in such a way that the total volume of the layer remains constant, with a corresponding increase in the upper and lower surface area of the layer (Figure 2). However, this may be somewhat pedantic, as even experts sometimes refer to vertical "compression" of the ice (i.e., Cuffey and Paterson, 2010, p. 357).

Annual Layer Thicknesses: Creation versus Uniformitarian Expectations

Uniformitarian age-depth models for thick ice sheets often make the simplifying assumption that the height of the ice sheet is constant, or nearly constant (Cuffey and Paterson, 2010, pp. 617–620). For instance, in his preliminary estimation of the amount of layer thinning in an ice sheet, Nye (1963, p. 786) implicitly assumed that the height of the ice sheet H is constant. Dansgaard and Johnsen (1969, p. 216) also made this assumption. At first glance, this does not seem to make sense. Regardless of whether one believes the ice sheet is thousands or millions of years old, its starting thickness *had* to be zero!

However, both creationists and uniformitarians agree that thick ice sheets can form relatively quickly, on the order of thousands or tens of thousands of years (Paterson, 1980, p. 40; Wilson, Drury, and Chapman, 2005). Not too surprisingly, the actual number is strongly dependent upon precipitation rates: the higher the precipitation rates, the less time is needed for the formation of the ice sheet. If one believes the ice sheet to be millions of years old, then one can *legitimately* simplify the calculations by treating the height of the ice sheet as constant. Any error introduced by ignoring the thousands of years for the ice sheet to grow to its steady-state height will be negligible in light of the ice sheet's multi-million-year history. Of course, if the ice sheet is just thousands of years old, then the time for the ice sheet's formation cannot be neglected.

Secular age-depth models predict much more thinning of the ice with depth than do young-Earth models, and some reflection upon Figure 1 shows why. In the absence of melting, all the annual layers are preserved in the ice, even if they cannot all be distinguished visually. In that case, the average thickness of an annual layer is simply the height of the ice sheet divided by the number of years it has been in existence. Naturally, subdividing the ice sheet height into millions of horizontal slices yields a much thinner average annual thickness than if you divided it into just 4,500 horizontal slices.

Although obtaining the average layer thickness is trivial, obtaining the true annual thicknesses as a function of depth is considerably more difficult, because these thicknesses depend upon the vertical strain rate (discussed below), which generally varies with time and depth (Cuffey and Paterson, 2010, pp. 614–617).

Hence, there is a need to be able to calculate the true thickness of an annual layer of ice in a creation model, in order to make comparisons between secular and creationist expectations. Oard (2005, p. 45) has already published some very rough estimates for the GRIP core in central Greenland (Figure 3), but there is a need to obtain more rigorous estimates for these thicknesses.

Creationists have already noted (Oard, 2005; Hebert, 2018b) that both the thicknesses and frequencies of tephra (volcanic ash and debris) layers in the deep ice cores make more sense in a creationist framework than in a uniformitarian one. Finding better estimates for annual layer thicknesses may help demonstrate other ways in which the creation model makes better sense of the data than do uniformitarian models.

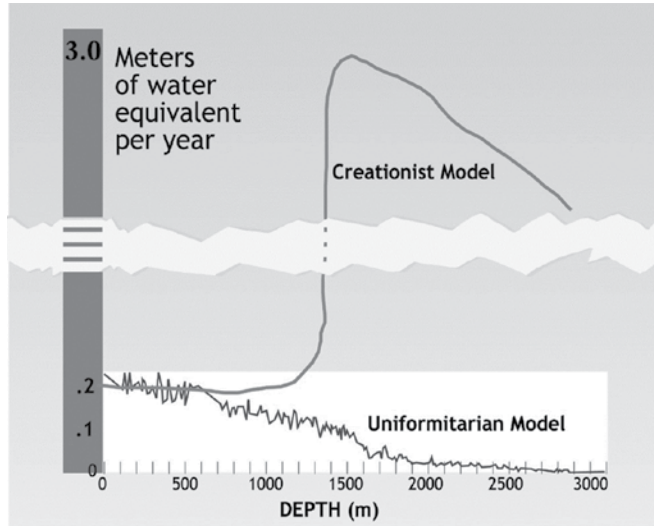

Figure 3. Rough estimates by Oard of annual layer thicknesses (in water equivalent) within Greenland's GRIP ice core, based upon uniformitarian and creationist expectations.

Governing Equation

In the absence of melting, the governing equation, at the divide location, for the growth rate of the ice sheet is:

$$\frac{dH}{dt} = \dot{b}(t) - w_s(t) \quad (1)$$

In Equation (1), H is the height of the ice sheet at the location of the ice divide, t is the time since the ice sheet began forming, and dH/dt is the time rate of change of the ice sheet divide height. $\dot{b}(t)$ is the accumulation rate at the surface of the divide (in meters of ice equivalent per year). $w_s(t)$ is the downward vertical speed of the ice sheet surface at the divide location, also in meters per year. In this discussion we will use z to represent vertical distance (in meters) above bedrock.

When the surface accumulation rate $\dot{b}(t)$ is greater than the downward vertical speed $w_s(t)$ of the ice sheet surface, the thickness of the ice sheet grows, so that the derivative in Equation (1) is positive. If the accumulation rate is equal to the downward surface speed, the derivative is equal to zero, and the ice sheet height remains constant in a "steady state" condition. Finally, if $\dot{b}(t)$ is less than $w_s(t)$, the height of the ice sheet divide will decrease.

Since the underlying bedrock prevents the bottom surface of the ice sheet from moving downward, one boundary condition is that the vertical velocity w is always equal to zero at height $z = 0$. The spatial derivative of vertical velocity is the vertical strain rate (Cuffey and Paterson, 2010, p. 615):

$$\frac{\partial w}{\partial z} = \dot{\varepsilon}_{zz}(t,z) \tag{2}$$

As we shall see later, the vertical strain rate is also (and is actually defined to be) the fractional change in thickness of an ice layer per unit time. In general, the strain rate at the divide is a function of both height z above bedrock and time t. Pioneering glaciologist J.F. Nye developed a simple thinning model (Nye, 1963) in which he assumed that the downward vertical speed increases linearly from zero at bedrock to a maximum value $w_s(t)$ at the surface. Or equivalently, the vertical velocity w has a maximum value of zero at the base, and this velocity decreases linearly with increasing z to a minimum value $-w_s(t)$ at $z = H(t)$. In this case, the strain rate is a function of time but not of depth.

$$\frac{\partial w}{\partial z} = \dot{\varepsilon}_{zz}(t) = -\delta(t) = -\frac{w_s(t)}{H(t)} \tag{3}$$

Vardiman's model is simpler still in that it treats the vertical strain rate $-\delta$ as a constant in both time and space. Not too surprisingly, vertical stress is greater at greater depths due to the greater weight of the overlying ice. However, the magnitude of the vertical strain rate is zero at a frozen base. For a frozen base, with horizontal velocity always zero at $z = 0$, the vertical strain rate must also be zero at $z = 0$, since the divergence of the ice velocity is zero (next section). Vertical strain rate is often approximated (Johnsen and Dansgaard, 1969) as a piecewise function whose magnitude increases from zero linearly up to some intermediate depth, and then remains constant all the way up to the surface. The strain rate also depends on time t. Future creation ice sheet models need to take into account space and time variations in the vertical strain rate.

Mass Balance

As implied by Equation (1), two competing processes influence the change dH in ice sheet height within a given time interval dt. The accumulation rate $\dot{b}(t)$ causes H to increase, but the thinning of underlying layers causes H to decrease.

Because we are treating the ice as incompressible, the divergence of ice velocity must everywhere be equal to zero. The ice core has a constant radius R and a cross-sectional area of πR^2 (Figure 4). Hence, any downward mass flux of ice (per unit time) at the top of the ice core must be balanced by the radially outward mass flux of ice (per unit time) occurring at $r = R$. For cylindrical coordinates, in the case of azimuthal symmetry, the divergence of the velocity (Griffiths, 1989, inside cover) is

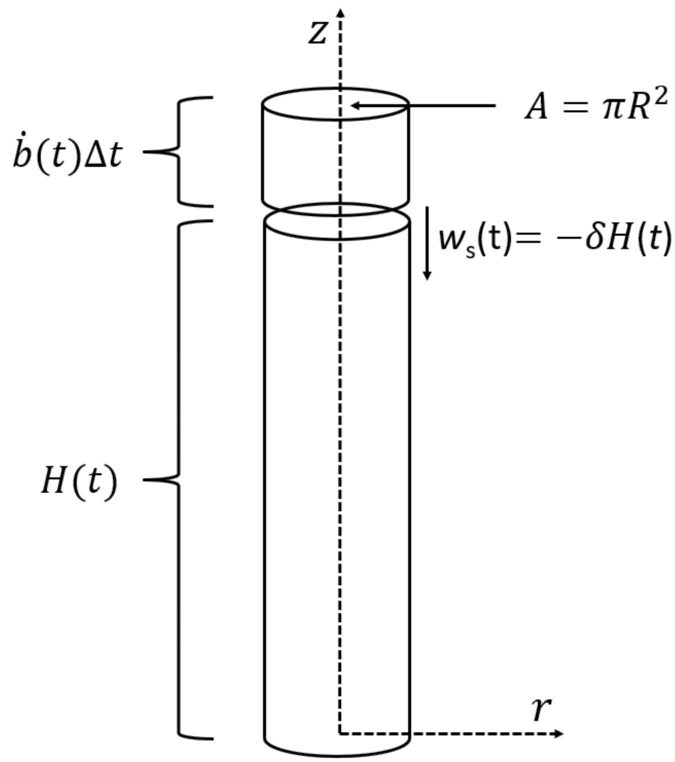

Figure 4. Diagram of an ice core of height H and radius R, at time t, just before the next layer of annual accumulation is deposited. The ice at $z = H(t)$ is moving downward with a speed of $\delta H(t)$. The next layer of ice accumulation will be deposited shortly after time $t + \Delta t$, after the already-deposited layers have been thinned.

$$\vec{\nabla}\bullet\vec{v} = \frac{1}{r}\frac{\partial}{\partial r}\left(rv_r\right) + \frac{\partial w}{\partial z} = 0 \tag{4}$$

Since the gradient of w is $-\delta$, we can solve Equation (4) to obtain the radially outward velocity of the ice:

$$v_r = \frac{\delta r}{2} = \frac{w_s(t)r}{2H(t)} \tag{5}$$

Note that, although $w_s(t)$ and $H(t)$ both depend on time, their ratio, δ, does not. The downward rate of mass flow (per unit time) into the cylinder's top face (Figure 4) is

Downward Mass Flow =

$$w_s(t)\rho A = [\delta H(t)]\rho A = \rho\delta H(t)\left(\pi R^2\right) \tag{6}$$

The radially outward rate of mass flow (per unit time) is the radial velocity evaluated at $r = R$, multiplied by the ice density ρ and the surface area of the cylinder's side:

Outward Mass Flow =

$$\rho\frac{\delta R}{2}[2\pi RH(t)] = \rho\delta H(t)\left(\pi R^2\right) \tag{7}$$

As expected, these two quantities are equal. In fact, mass balance is implied by Equation (1), since Equation (1) can be converted into a mass balance equation by multiplying both sides of the equation by ρA: the net change in mass of the ice core per unit time is the difference between the mass accumulation rate at the surface and the total mass outflow rate at the side of the core.

Vardiman's Model

Here we use Larry Vardiman's analytical ice sheet model (1993) and a simple computer code to estimate layer thicknesses in Vardiman's model. Vardiman wrote my Equation (1) as

$$\frac{dH}{dt} = \frac{\lambda(t)}{\tau} - \delta H \tag{8}$$

Here $\lambda(t)$ is the thickness of ice accumulation that occurs in a time increment $\tau = 1$ year. Since τ is always equal to 1, it may seem redundant to include it here, but we do so for purposes of dimensional consistency, as well as consistency with Vardiman's notation. As noted earlier, the vertical strain rate at the divide, $-\delta$, is constant in both time and space. By comparison with Equation (1), we see that Vardiman has implicitly assumed that the downward surface speed at the divide is always proportional to the height of the ice sheet.

Equation (8) may be rewritten as

$$\frac{dH}{dt} + \delta H = \frac{\lambda(t)}{\tau} \tag{9}$$

Vardiman reasonably assumed that the post-Flood ice accumulation rate would start very high and then smoothly decay over time to today's "slow and gradual" average accumulation

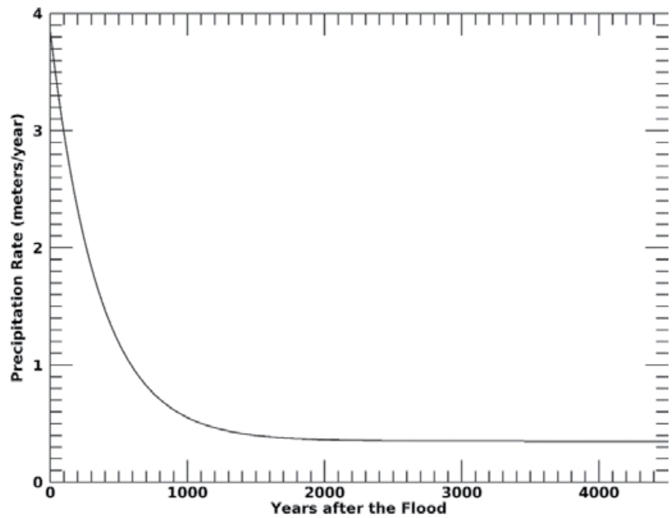

Figure 5. Vardiman's model for post-Flood ice accumulation at Greenland's Camp Century core, using parameter values of $\Phi = 10.0$, $\Psi = 350$ years, $H_\infty = 1370$ meters, $\lambda_H = 0.35$ meters, and $\delta = 2.55\times10^{-4}$ meters^{-1}.

rates. Therefore, he modelled the accumulation rate as a decaying exponential (Figure 5):

$$\dot{b}(t) = \frac{\lambda(t)}{\tau} = \frac{\lambda_H}{\tau}(\Phi e^{-t/\Psi} + 1) \tag{10}$$

Here λ_H/τ is today's accumulation rate, in meters of ice equivalent per year. Φ is a dimensionless number greater than 1 used to "scale up" the amount of Ice Age precipitation compared to today's average annual value. Likewise, t is the time since the ice sheet began forming, and Ψ is the e-folding time, the time for the initial post-Flood accumulation rate to drop to 37% of its original post-Flood value.

After including the expression for the accumulation rate, Equation (9) may be solved with the mathematical "trick" (Zill, 1989, p. 66) of multiplying both sides of the equation by an "integration factor," in this case, $e^{\delta t}$. This makes the left-hand side of Equation (9) the exact derivative of $He^{\delta t}$. Integration (see Vardiman, 2001 for details) yields this expression for $H(t)$:

$$H(t) = \frac{\lambda_H}{\tau\delta}\left(1 - e^{-\delta t}\right) + \frac{\Phi\lambda_H}{\tau\left(\delta - 1/\Psi\right)}(e^{-t/\Psi} - e^{-\delta t}) \tag{11}$$

Vardiman then imposed the boundary condition that the height of the ice sheet equal its steady state height H_∞ when $t \rightarrow \infty$. In particular, he used the Camp Century ice core from Greenland, whose current height is 1,370 meters and which would presumably have a frozen base (no melting) due to the low temperature near bedrock (Dansgaard and Johnsen, 1969, p. 216). This results in

$$H_\infty = \frac{\lambda_H}{\tau\delta} = 1370 \text{ m} \tag{12}$$

Using today's average accumulation rate of 0.35 meters/year, this allows us to solve for the parameter δ:

$$\delta = \frac{\lambda_H}{(1370 \text{ m})\tau} = 2.55\times10^{-4}\text{year}^{-1} \tag{13}$$

Figure 6 is a graph of $H(t)$ versus time since the Flood, using values of $\Phi = 10$, $\Psi = 350$ years, and a total time since the Flood of 4,500 years. The height of the ice sheet at $t = 4{,}500$ years is 1,362 meters, not quite the full 1,370-meter height obtained at $t = \infty$.

In passing, I should note that there appears to be an error in Vardiman's (2001) expression (his Equation 4.20, p. 49) for the vertical velocity. This is not intended as a criticism, but merely as a "guidepost" for future researchers in this area. Using my notation, his expression is

$$w(z) = -\delta \frac{z}{H_\infty} H(t) \tag{14}$$

Based on Equations (3) and (13), and the boundary condition that $w(z = 0,t) = 0$, it seems this expression should actually be

$$w(z) = -\delta z = -\frac{\lambda_H}{\tau H_\infty} z, \quad z \le H(t) \tag{15}$$

Note (Figure 4) that at $z = H(t)$, the downward velocity is $-\delta H(t)$, as indicated by Equation (15). Note also that the downward velocity is $-\lambda_H/\tau$ when $z = H_\infty$, as this is the downward velocity needed for the ice sheet to remain in steady state.

Numerically Obtaining Annual Layer Thicknesses

A simple computer code may be used to obtain the thickness of each annual layer at any given time, assuming that $H(t)$ and $\dot{b}(t)$ both are already known. I used the Interactive Data Language (IDL) to write my code. In this example, we use Vardiman's values for these quantities, calculated using the same parameter values used to produce Figures 5 and 6. The code uses double precision values for both constants and arrays to ensure numerical accuracy. However, in order to demonstrate the method in Figures 7–10, I do *not* practice rigorous adherence to significant figure rules. The reason for this is that rigid adherence to these rules, combined with rounding errors, will obscure the relationships between the numbers in Figures 7, 9, and 10, making it more difficult for the reader to follow the methodology I used.

The code establishes two arrays, each with 4,501 elements, each element corresponding to a particular time t since the Flood (and inclusive of $t = 0$). These arrays (Figure 7) give the values of $\dot{b}(t)$ and $H(t)$ at 4,501 particular times since the Flood, with an increment $\Delta t = t = 1$ year between each of these times. We are using a discrete numerical process to simulate a continuous process, so to keep our thinking straight, it is important to explicitly note the *precise* meaning of these $H(t)$ values. Each value of $H(t)$ is the total ice-core-divide height at the very start of a time increment, just *before* any of that year's ice accumulation has been deposited. Furthermore, we can imagine the entire year's worth of accumulation occurring just a tiny fraction dt of a year later. After the ice layer has been deposited, the thinning process begins and occurs continuously throughout Δt until the start of the next time interval.

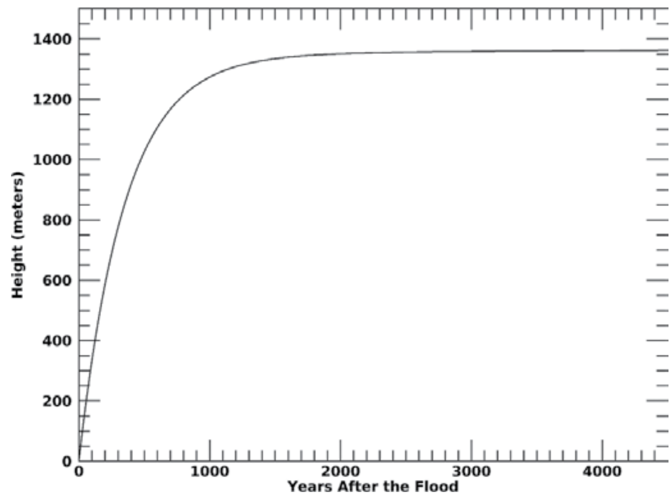

Figure 6. Height of the Camp Century (Greenland) ice sheet as a function of time since the Flood, using Vardiman's ice sheet model, with the same parameter values used to produce Figure 5.

Time t after Flood (in years)	Annual Precipitation $\dot{b}(t)$ (meters of ice equivalent/year)	Height of Ice Sheet $H(t)$ (meters)
4,500	0.35000913	1362.1697
4,499	0.35000915	1362.1677
4,498	0.35000918	1362.1657
4,497	0.35000920	1362.1637
⋮	⋮	⋮
3	3.8201282	11.500715
2	3.8300570	7.6780746
1	3.8400143	3.8445134
0	3.8500000	0.00000000

Figure 7. Vardiman's model is used to calculate the values of H and $\dot{b}(t)$ in advance for the 4,500 years since the Flood.

We also establish an array λ, with 4,500 elements, each element corresponding to the current thickness of each one of the 4,500 annual layers deposited since the Flood. Since no layers have yet been deposited at time $t = 0$, all the elements in λ are initially set to 0. These values will be updated iteratively as the code runs.

The infinitesimal vertical strain ε_{zz} of a layer of ice is defined as the fractional change in the thickness of that particular layer:

$$\varepsilon_{zz}(z,t) = \frac{d\lambda}{\lambda(z,t)} \tag{16}$$

Since the layer thins, $d\lambda$ is negative, as is the strain. Taking the derivative of both sides gives the strain rate, the fractional change in thickness per unit time (Cuffey and Paterson, 2010, p. 615):

$$\dot{\varepsilon}_{zz}(z,t) = \frac{d\varepsilon_{zz}}{dt} = \frac{1}{\lambda}\frac{d\lambda}{dt} \tag{17}$$

Separation of variables and integrating from time t to time $t+\Delta t$ yields

$$\lambda(t+\Delta t) = \lambda(t)e^{\left(\int_t^{t+\Delta t} \dot{\varepsilon}_{zz}(t)dt\right)} \tag{18}$$

In other words, the thickness of a given ice layer after thinning will be the thickness prior to thinning, multiplied by the exponential, which I am calling "the fraction of thickness retained." Since $\dot{\varepsilon}_{zz}(t)$ is inherently negative (or zero), this fraction will always be less than or equal to 1, which implies that the thickness of the layer after thinning cannot be greater than it was before thinning. Note that Equation (18) is applicable to all 4,500 annual layers of the ice sheet, even if a layer has not yet been deposited. In that case, the fraction is simply multiplied by 0 meters, leaving the original thickness of that (not yet deposited) layer as still 0.

Two things should be noted regarding Equations (17) and (18). First, since the strain rate does not depend on height z in Vardiman's model, and because the total thickness of the ice sheet H is equal to the sum of the thicknesses of the individual layers, it is also true that

$$\text{Fraction of thickness retained} = \frac{\lambda(t+\Delta t)}{\lambda(t)} = \frac{H(t+\Delta t)}{H(t)} \tag{19}$$

Second, remember that conceptually we are imagining that the entire year's worth of ice accumulation occurs just a tiny fraction of a year *after* the start of the time interval. Since the thinning process does not begin until the year's accumulation has been deposited, we should, when calculating the fraction of retained thickness (Figure 8), replace $H(t)$ with $H(t) + \dot{b}(t)\tau$, since this is the true thickness just before the thinning process begins:

$$\text{Fraction of thickness retained} = \frac{\lambda(t+\Delta t)}{\lambda(t)} = \frac{H(t+\Delta t)}{H(t)+\dot{b}(t)\tau} \tag{20}$$

Demonstration of the Method

As noted earlier, at the start of the program ($t = 0$), all of the 4,500 cells in the λ array are assigned a value of zero, indicating that all 4,500 annual layers currently have a thickness of 0

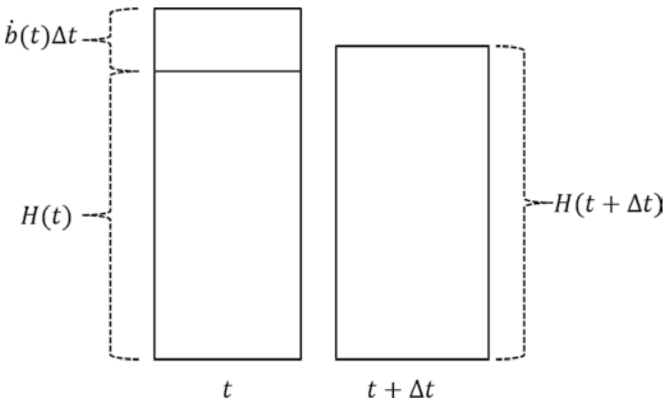

Figure 8. Due to thinning, the height of the ice sheet at time $t + \Delta t$ will be less than the sum of the previous height and the thickness of ice that was deposited just after t. Note that $\Delta t = \tau = 1$ year.

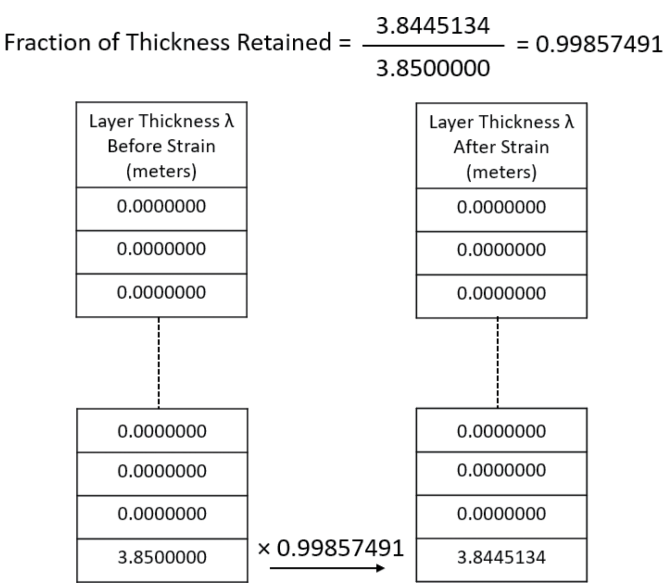

Figure 9. Illustration of how Equation 20 and Figure 7 are used to calculate the thickness of the first annual layer after thinning.

meters. All 4,501 values of $H(t)$ and $\dot{b}(t)$ have been calculated from Vardiman's model and stored in the appropriate arrays (Figure 7).

During the first year, exactly 3.85 meters of ice is deposited (Figure 7). Conceptually, we can imagine that all 3.85 meters of this ice is deposited at an infinitesimal time dt after $t = 0$. This is the layer thickness λ before the thinning that occurs

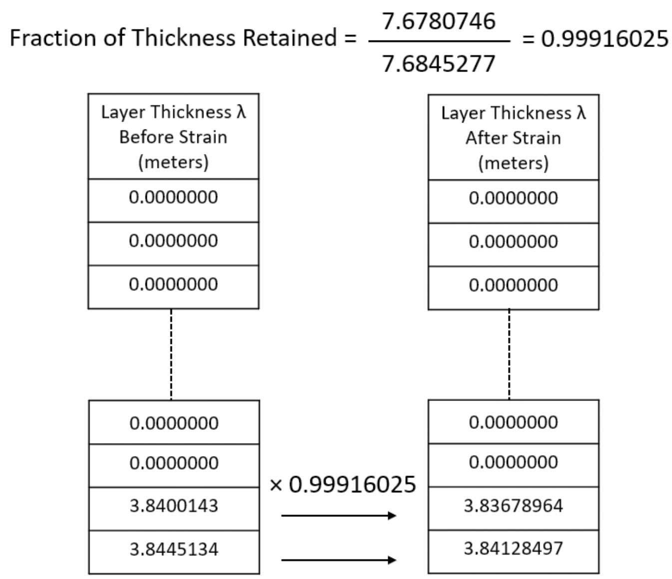

Figure 10. Illustration of how Equation 20 and Figure 7 are used to calculate the thicknesses of the first two layers, after the second annual layer has been deposited and both of the two layers have been thinned.

between $t = 0$ and $t = 1$. The fraction of the layer thickness retained will be

$$\begin{aligned}\text{Fraction of thickness retained} &= \frac{H(1)}{H(0) + \dot{b}(0)\tau} \\ &= \frac{3.8445134 \text{ m}}{0 \text{ m} + 3.8500000 \text{ m}} \\ &= 0.99857491\end{aligned} \tag{21}$$

Note that the first values of $H(t)$ and $\dot{b}(t)$ are designated by the index $i = 0$, not $i = 1$, because IDL requires its array indices to start at 0, rather than 1. From Equations (18) and (20), and Figure 9, we see that the thinned thickness of this first layer will be

$$\begin{aligned}\lambda_{after}(1 \text{ year}) &= \lambda_{before} \times 0.99857491 \\ &= 3.8500000 \text{ m} \times 0.99857491 \\ &= 3.8445134 \text{ m.}\end{aligned} \tag{22}$$

In this particular case, the starting value of λ is equal to $\dot{b}(t)\tau$. This will always be true for the uppermost layer, which

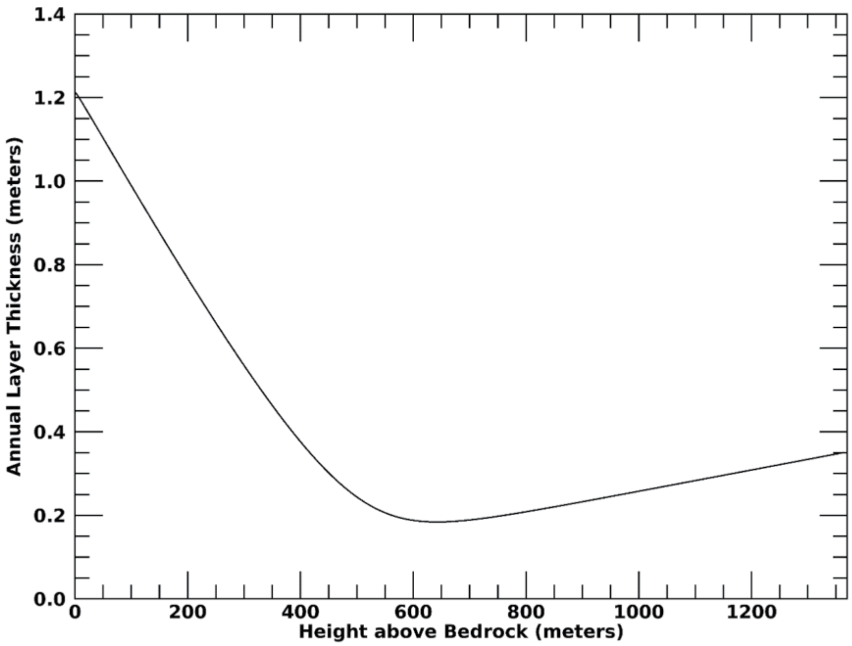

Figure 11. Resulting Camp Century annual layer thickness as a function of height above bedrock. The sum of the thicknesses of all 4,500 layers is 1,362.1697 meters, the height of the ice sheet at t = 4,500 years (see Figure 7).

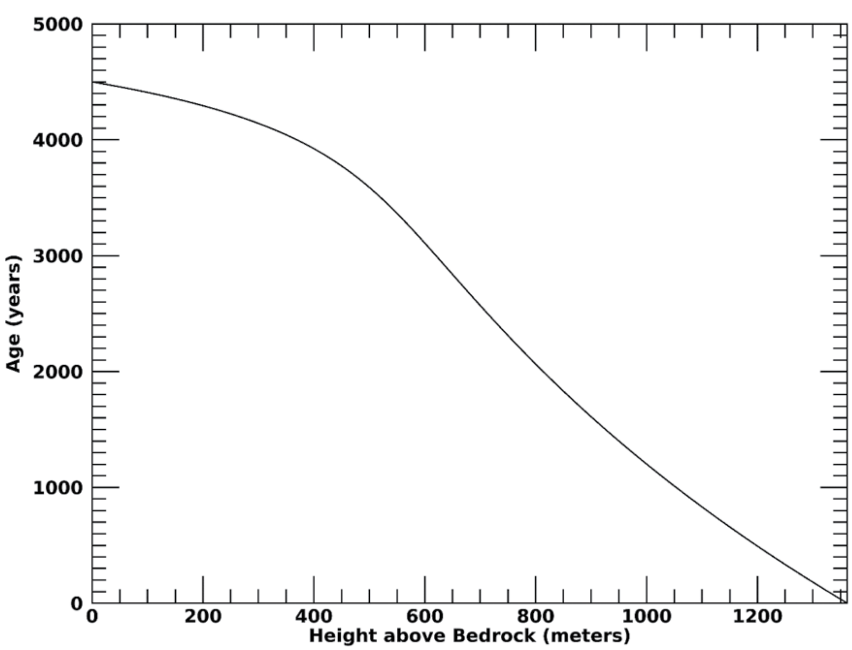

Figure 12. Age of Camp Century annual layers as a function of height above bedrock.

has just been deposited, but it is not true for the layers below it. Note that the ending thickness of this first layer (Figure 9) is equal to the height H (Figure 7) at the start of the next time increment (t = 1 year), as it should be.

This process is then repeated. A second layer $\dot{b}(1)\tau$ of thickness 3.8400143 meters (Figure 10) is deposited. Equations (18) and (20), together with the updated thickness $\lambda(0) = H(1)$ of the bottom layer, gives us a "fraction of thickness retained" of

$$\text{Fraction of thickness retained} = \frac{H(2)}{\left(H(1)+\dot{b}(1)\tau\right)} = \frac{H(2)}{\left(\lambda(0)+\dot{b}(1)\tau\right)} \tag{23}$$

Substituting the appropriate values into Equation (23) yields a fraction of 0.99916025. Multiplying both current thicknesses of these two layers by this fraction gives their new thicknesses (Figure 10). Note that the sum of these two new thickness is 3.84124897 m + 3.83678964 m = 7.6780746 m, which is the height of the ice sheet at t = 2 years (Figure 7), as it should be. The process is repeated until all 4,500 layers have been filled. As a consistency check, the height of the ice sheet at t = 4,500 years is 1,362.1697 meters, which is also the final sum of all 4,500 individual layer thicknesses.

Results

Figure 11 is the resulting graph of annual layer thickness as a function of height above bedrock. Note that the annual layer thickness at the top of the ice sheet is ~0.35 meters, as it should be, since the present-day accumulation rate is ~0.35 meters per year, and the uppermost layer has undergone negligible thinning. The bottommost layers have been thinned quite a bit, but are still much thicker than one would expect from a uniformitarian model.

That the minimum layer thickness in Figure 11 occurs, not at the very bottom of the core, but at an intermediate depth, makes physical sense. The much higher precipitation rates shortly after the Flood will tend to make the deepest annual layers much thicker than those at the top of the core. On the other hand, the deepest layers have also been thinning for the longest amount of time. These two competing processes will cause the minimum thickness to occur, not at the very bottom of the ice sheet, but at an intermediate depth, in this case about 600 meters above bedrock.

Since we know the exact time at which each layer was deposited, it is also possible to obtain the age of the ice as a function of height above bedrock (Figure 12).

An Alternate Method

We may simplify Equation (18) to obtain another expression for the final annual layer thickness for a layer having an age equal to (4,500 – t) years. Since $-\delta$ is constant, it may be "pulled out" of the integral:

$$\begin{aligned} \lambda &= \lambda_0 e^{-\delta \times age} \\ &= \dot{b}(t)\tau \cdot e^{-\delta \times (4500-t)} \end{aligned} \quad (24)$$

In words, the final thickness is the original thickness of the layer at the time it was deposited, multiplied by an exponential thinning factor that depends on the layer's age. Use of Equation (24) yields final layer thicknesses that are in very good agreement with Figure 11.

Discussion

These results should not be taken too seriously, as they were derived from the assumption that downward surface speed at the divide is proportional to the height, which is unrealistic (Vardiman himself acknowledged that his work was a preliminary effort). Nevertheless, it illustrates some important ice core basics. Hopefully this will become the springboard for more sophisticated creation ice sheet models in the future. The Mahaffy (1976) model may be a good place to start, as it allows for a time-varying accumulation rate and does *not* assume steady-state, as do most uniformitarian ice sheet models.

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