Towards a Rigorous Methodology for Quantifying Truncation of Anticlines: A Case Study at Mount St. Helens

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Abstract

The regional-scale erosion of hundreds to thousands of meters of stratigraphy remain one of the most pivotal yet debated arguments within Flood Geology. Despite the many previous studies of erosion of anticlines as one illustration of regional-scale erosion, no rigorous mathematical model has been proposed for systematic and repeatable modeling of fold surfaces for erosion estimates in data scarce locations. As such, this study proposes a boundary-value problem approach for modeling symmetric and non-verging fold systems. Applied to folds in the Mount St. Helens region, the model performed well in describing characteristics of the half-wavelength of the fold system. The modeled surface resulted in a calculation of 6.16 km of vertical relief eroded from the current topography, a value that could be increased to 10.1 km when transferring the modeled surface to the outermost observed fold surface. Site-specific geology suggests an additional 1 to 4 km of stratigraphy may have rested atop this modeled surface. This application of boundary-value problems represents a promising technique to systematically reconstruct fold systems for erosion estimates. The approach requires minimal inputs that are easily acquired from geologic maps although this limits its application to approximately symmetric and non-verging fold systems. Even so, this technique represents a first step towards developing an easily deployable yet rigorous approach to model fold systems for repeatable and consistent erosion estimates.

Key Words: Differential Geometry, erosion, fold system reconstruction, Mount St. Helens

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Introduction

Regional-scale erosional patterns remain one of the most central yet debated arguments in Flood Geology. The removal of hundreds to thousands of meters of stratigraphy from the surface of a landscape remains difficult to explain within naturalistic geology and its limited erosional power (e.g., Froede, 2004; Oard, 2008; Matthews and Oard, 2015; Isaacs, 2020). Instead, these authors argue that examples of extreme erosion are evidence of past, high-energy erosion on a regional scale consistent with the scales proposed within a Biblically-founded geologic framework (Reed et al., 1996). Although one of the signature arguments of Flood Geology, how such examples of regional-scale erosion fit into the Biblical timeline remains a topic of debate (e.g., Holt, 1996; Whitmore, 2006; Oard, 2017). Some researchers argue that this erosion is a defining characteristic of recessive processes during the late-stage Genesis Flood (Oard, 2017), while others explain many surficial examples of regional-scale erosion as resulting from post-Flood catastrophism while the world tended towards a state of quasi-equilibrium before the post-Flood Ice Age (Whitmore and Garner, 2009; Whitmore, 2013). Despite this debate being pivotal in the overarching Flood/post-Flood controversy, no work has been done to develop a systematic, mathematically rigorous approach to calculating truncation of anticlines despite their frequent mention in this literature.

The lack of methods for reconstructing anticlines is in part due to the continual challenge that modeling structures pose in industry and academia (Carrera et al., 2009). Traditionally, fold systems are reconstructed using structural contours, which are defined by the intersection of geologic units and topographic contours. However, this can be extremely tedious by hand and requires sufficiently exposed bedrock geology. Alternatively, mathematical models have been developed for interpolating between points where X, Y, and Z values are known. These models generally can only be applied to thoroughly mapped locations with abundant data such as GPS points, cores, and seismic imaging, which are unavailable for many geologic structures (Carrera et al., 2009; Hou et al., 2023; and references therein). In the absence of alternative methods for data scarce structures, Flood geologists have employed various empirical back-of-the-envelope approaches for estimating the scale of erosion (e.g., Oard, 2008; Matthews and Oard, 2015; Isaacs, 2020).

Given the integral nature of this question for other overarching questions in Flood Geology, it is important to develop consistent and mathematically rigorous techniques for estimating regional-scale erosion in data scarce locations. To that end, this paper investigates an application of differential geometry to develop a simplified mathematical model to reconstruct truncated symmetrical anticlines with vertically oriented axial planes (that is, non-verging). This approach is illustrated with a case study in the Mount St. Helens region to estimate the level of vertical erosion (truncation) of the Lakeview Peak Anticline stratigraphy.

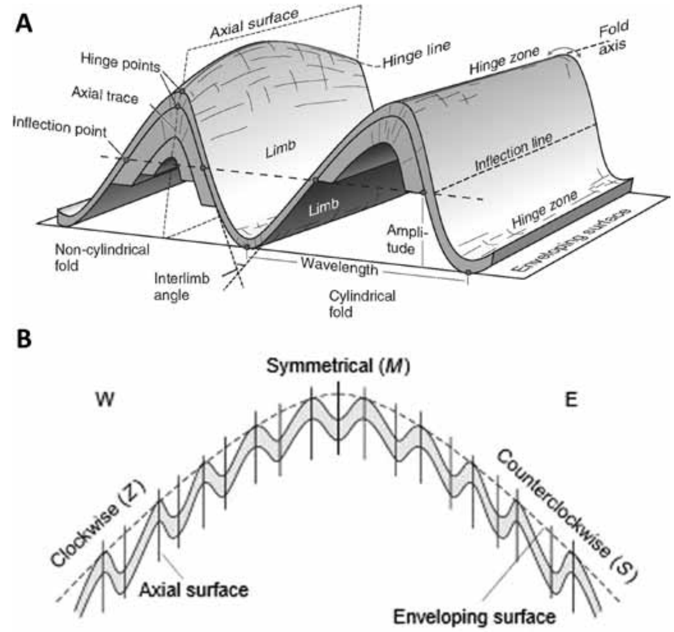

Figure 1. A) Anticline-syncline fold systems are commonly conceptualized as cylindrical curves that are formed from the shortening and compression of what were once planar beds. Note that beds are folded parallel to each other to create a sequence of parallel cylindrical surfaces where a single function could be used to describe the shape of each individual bed. The axial surface is the plane that intersects the hinge points (peak) of each folded bed in an anticline. When a series of parallel axial surfaces are non-vertical (e.g., slanted to the west), it is referred to as vergence (such as "westward verging folds"). Figure after Fossen (2016). B) Folds may also be combined into composite structures like anticlinoria, which are comprised of many smaller parasitic folds along the overarching enveloping surface of the composite structure. Note that axial surfaces are vertical meaning that this fold has no vergence. Figure after van der Pluijm and Marshak (2004).

Geologic Applications for Differential Geometry

As an application of calculus, differential equations are frequently used when describing infinitesimally small

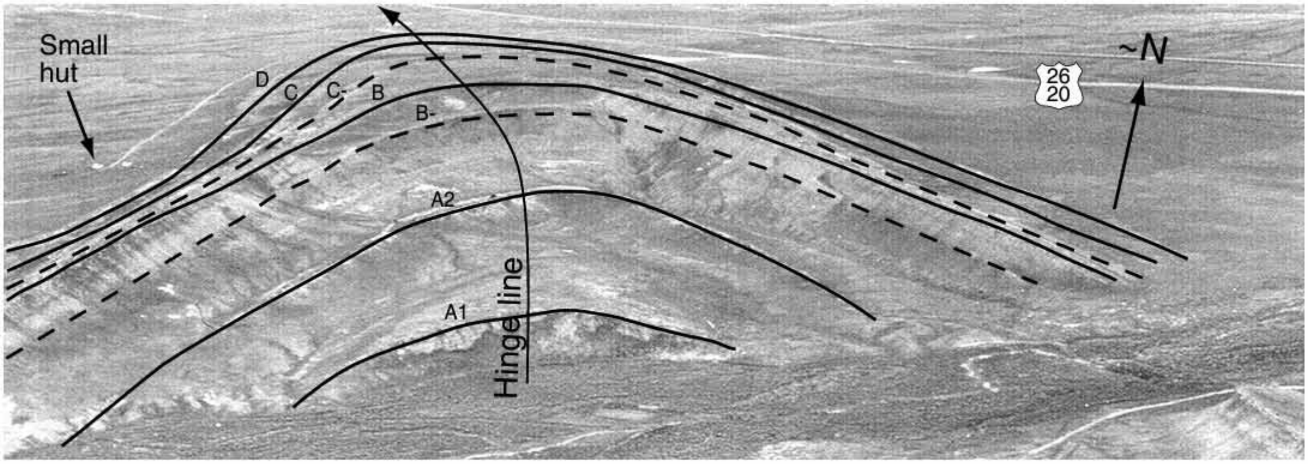

Figure 2. Oblique aerial photograph of the Emigrant Pass anticline in Wyoming. Tracing by Bergbauer and Pollard (2004) used to reconstruct the anticline shape using differential geometry. Figure from Bergbauer and Pollard (2004).

changes in one or more properties necessary to model a natural phenomenon. As such, geomorphology frequently employs differential calculus to derive equations to describe processes such as stream flow and isostasy or to develop landscape evolution models (Pelletier, 2013; Bierman and Montgomery, 2014). Differential equations need not be used only for deriving quantitative relationships between variables but can also be used to describe geometry, such as curvilinear features such as anticline-syncline fold systems. Resulting from the compression of originally planar beds into a curved geometry, fold systems are frequently illustrated as sine-cosine waves and assumed to be cylindrical. As such, Fourier systems, or the sum of an infinite number of sine and cosine functions, is the basis for describing the general shape of fold geometries (Fossen, 2016, p. 258). Hudleston (1973) used a simplified Fourier system to describe fold geometry as approximating the curve:

$$f(x) = b_1 \sin(x) + b_3 \sin(3x) + b_5 \sin(5x) \ldots$$

Where coefficients b_n are unique to the folds being described. Using this Fourier system, folds can be described of a variety of geometries including when their axial plane is nonvertical (that is, verging folds; see Figure 1) though not when they are recumbent. When folds are harmonic, beds are folded parallel to each other so that the function that describes one bed can be applied to all other parallel beds (Fossen, 2016, pp. 258–259). However, folds do not always follow these generalized systems, as has been illustrated at Emigrant Pass in Wyoming (Figure 2) (Bergbauer and Pollard, 2004). Using GPS points to constrain the geometry, Pollard and Fletcher (2005, pp. 116–119) apply best-fit lines through eight cross-sections to illustrate the approximate shape of the fold (as shown in Figure 3). Although asymmetrical, these shapes remain reminiscent of sine-cosine waves that can be modeled by differential geometry.

A Boundary-Value Approach to Modeling Symmetrical, Non-Verging Anticlines

The continual challenge in geomathematical modeling is balancing the precision of the technique and the availability of data. For instance, most geologic mapping projects will not include a series of GPS points for constraining differential geometric models of fold systems. However, regional geologic maps will frequently record several key pieces of information when mapping fold systems:

1. Strike and dip measurements of bedding planes along the fold system.
2. Approximate elevation of strike and dip measurements as shown plotted on a topographic map.
3. The axial trace (that is, the axial plane of the fold intersecting the land surface) calculated through a stereonet analysis of collected strike and dip measurements of folded units.

With this information, the approximate wavelength between interpreted folds can be known as well as the axial plane of wave minima (synclines) or maxima (anticlines)

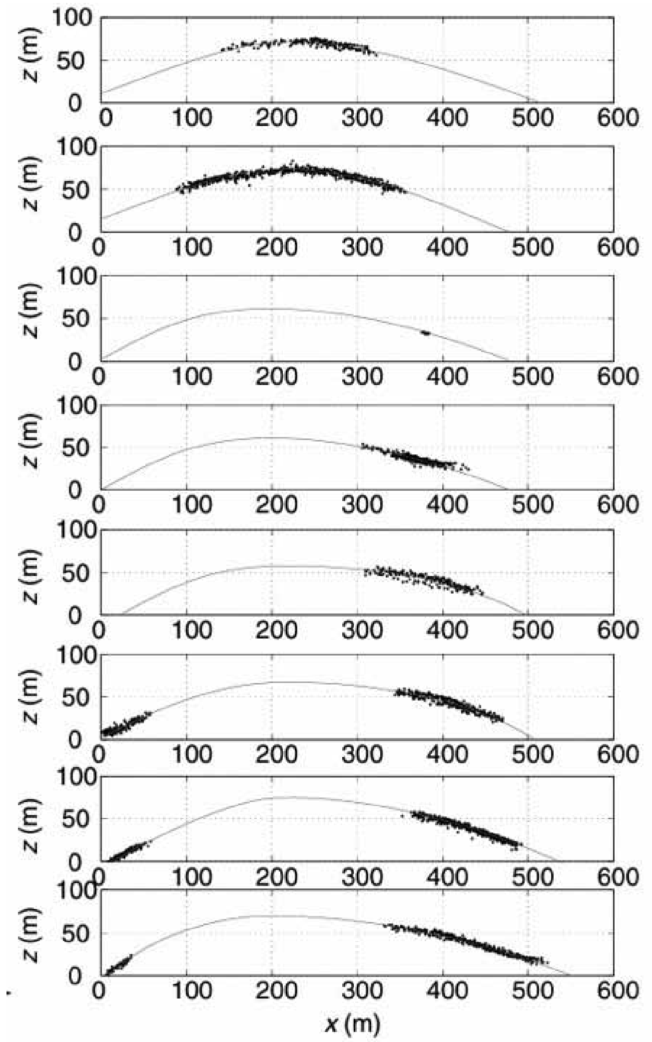

Figure 3. Eight profiles of the Emigrant Pass anticline in Wyoming that show best fit lines through GPS control points to constrain the modeled geometry of the anticline. Note that, although asymmetric, the modeled surfaces are still reminiscent to sine-cosine waves. Figure from Pollard and Fletcher (2005, pp. 116–119).

and a series of points of known location and bedding slope. This provides the constraints for a boundary-value problem, a specific category of ordinary differential equations where a point of known position and slope is needed to reconstruct the curvilinear geometry. We can define one generic boundary-value problem differential equation as:

$$y'' + a^2 y = 0$$

Where a is a coefficient that captures "half wavelength" $\lambda_{1/2}$ or the distance between adjoining maximum (peak or anticline) and minimum (trough or syncline):

$$a = \left(\frac{\pi}{\lambda_{\frac{1}{2}}}\right)$$

In this system, $y(x)$ is the function that describes the fold geometry of interest and where $y(0)$ is the elevation of the strike and dip measurement of interest (that is, the position of a point of known slope). The first differentiation $y'(0)$ is equal to the slope of the strike and dip measurement of interest. With a, $y(0)$, and $y'(0)$ known, this system can be solved by hand or with a variety of online, ordinary differential equations (ODE) calculators. The generic solution is:

$$y = C_1 \cos(ax) + C_2 \sin(ax)$$

Solving for coefficients C_1 and C_2 using the initial conditions $y(0)$ and $y'(0)$ to create a system of equations will yield the function that describes the modeled fold surface. It is important to note the sign convention. Positive $y(0)$ and $y'(0)$ will be for systems where the nearest anticline is to the right and the nearest syncline is to the left. If the reverse is the case, the values of $y(0)$ and $y'(0)$ will be negative.

If the fold is harmonic, beds are parallel so that the function calculated for one bed surface can be moved vertically in space to fit to other beds in the sequence. This is important given that the modeled bed surface may not be the uppermost folded surface in the sequence. On flat terrain, this uppermost surface would be exposed in the center of the syncline in plan view, which may consequently have the least data given its minimal exposure. The model function, however, can be vertically translated in space until the model syncline axis coincides with the mapped syncline axis.

The model will work for the desired fold system assuming that the fold system is non-verging, symmetrical, and cylindrical. For some folds, this will be adequate to approximate the fold surface. The level of truncation (that is, the erosion of the highest original point of the fold to the current land surface) can be calculated as the difference between the elevation of the initial point $y(0)$ and the maximum elevation on the fold.

Case Study at Mount St. Helens

Although known for its 1980 eruption, Mount St. Helens is but the latest example of a long history of volcanism along the Cascadia Magmatic Arc (Cheney, 2014). Volcanic activity dominated the region beginning in the Eocene, result-

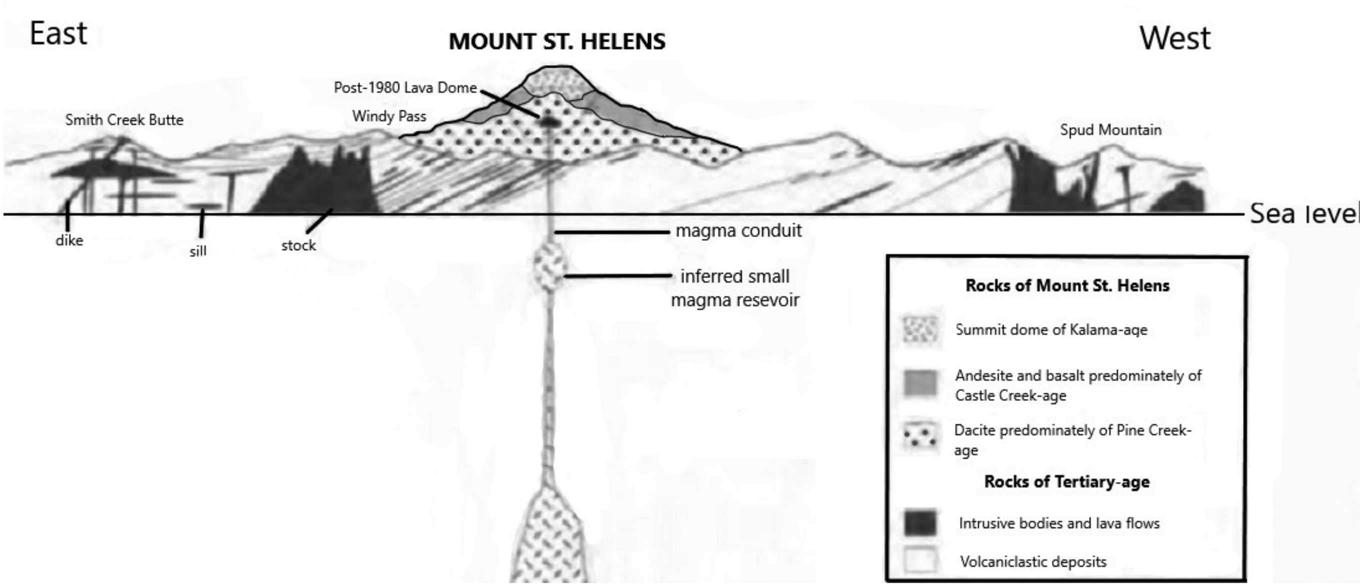

Figure 4. A cross section of Mount St. Helens and the underlying eastward dipping limb of the Lakeview Peak Anticline to the west and adjacent Pole Patch Syncline to the east. Modified from Pringle (2002).

ing in a record of explosive and effusive eruptive products even before the formation of Quaternary stratocones like Mount St. Helens (Evarts et al., 1987). Eocene to Miocene volcaniclastics and lavas are folded in a series of parasitic folds in the Cascade Anticlinorium, an anticlinal composite fold comprised of many smaller, parasitic anticlinal and synclinal folds from southern British Columbia to northern California (Cheney, 2016). Mount St. Helens straddles two of these parasitic folds: the Lakeview Peak Anticline to the west and the Pole Patch Syncline to the east (Figure 4) (Evarts et al., 1993; Evarts, 2001). Consequently, Mount St. Helens volcanic products overlay a series of Oligocene to Miocene volcanics that dip to the east (Figure 5).

Isaacs (2020) attempted to extrapolate the observed anticlinal surface to estimate the vertical relief that has been eroded from the region. By using trigonometry to describe a sine function, Isaacs calculated that the crest of the anticline was at least 7.85 km above the current landscape. However, this approach is a back-of-the-envelope method that assumes that the arbitrarily chosen sine function adequately models the fold system, which may or may not be the case. The previously described ordinary differential equation negates this problem by using specific geometric information to model the original anticlinal surface in a mathematically rigorous and repeatable way.

As shown in Figure 6 (Isaacs, 2020), the distance $\lambda_{1/2}$ between the Pole Patch Syncline and Lakeview Peak Anticline is 44.2 km. Slopes near the Smith Creek Butte, the point of known elevation and slope used in Isaacs (2020), vary between 20° and 25°. For $y'(0)$, we will choose 20° or 0.349 radians (rad) as it will assume a more gentle slope and thereby lower anticline crest elevation to give a minimum level of vertical truncation. As such, we can solve the system:

$$y'' + \left(\frac{\pi}{44200\text{ m}}\right)^2 y = 0, y(0) = -1130\text{m},$$
$$y'(0) = -0.349$$

Note that we are setting $y(0)$ and $y'(0)$ to negative values because the nearest syncline is to the east and the nearest anticline is to the west when facing north (see earlier description). Our resulting modeled function is:

$$y = -1130\cos\left(\frac{\pi}{44200}x\right) - 5120\sin\left(\frac{\pi}{44200}x\right)$$

This function is shown in Figure 7; notable geometric values that come out of this function are noted in Table I. The elevation difference between $y(0)$ and the peak of the modeled anticline is 6.16 km.

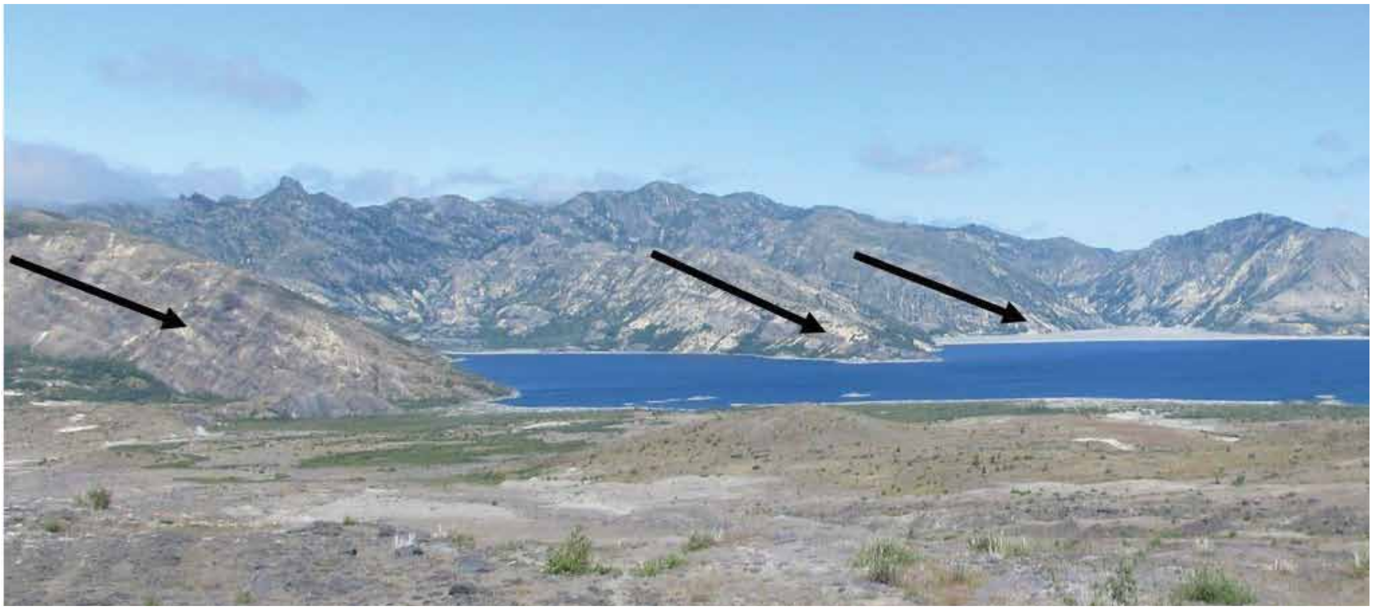

Figure 5. The deforestation along the northern regions of Mount St. Helens during the 18 May 1980 eruption left the eastwardly dipping formations (denoted by arrows) of the west limb of the Pole Patch Syncline strikingly visible around Spirit Lake.

Discussion

From this case study of Mount St. Helens, we can inspect the performance of the proposed differential equation for modeling anticlines and by extension the scale of regional erosion. As noted in Table I, The distance from $y(0)$ to the axis of the modeled syncline and modeled anticline is 14.3% and 10.7% from their mapped counterparts, respectively (when comparing to their measured distances in Figure 6). This illustrates that, despite the fold system not following the assumption of symmetrical and non-verging, the modeled function performs well in describing the constraints concerning the half-wavelength $\lambda_{1/2}$ of the mapped fold system.

The elevation difference between our initial point of interest and the peak of the modeled anticline is 6.16 km, implying that 6.16 km has been eroded from the anticline's crest to the current land surface. However, the point of interest is not on the outermost bed of the fold system. As discussed earlier, in harmonic fold systems the beds will be parallel to each other, allowing the function determined for one bed to be vertically translated to fit to other known beds in the sequence to show the uppermost exposed surface. Assuming an average elevation of 1130 m on the modern terrain, the modeled surface would be translated vertically 3.90 km to match the elevation of the uppermost known folded surface that is exposed in the Pole Patch Syncline (as illustrated in Figure 7). As such, the total estimated eroded relief reaches a value of 10.1 km.

Swanson (1992) noted that even the exposed stratigraphy of the Pole Patch Syncline may not have been the uppermost surface of the fold system. Due to the interpreted thermodynamic requirements of post-folding intrusives in the Pole Patch Syncline, Swanson believed that 1–4 km of stratigraphy had been eroded above what is currently the uppermost exposed fold surface, which would represent an additional 1–4 km of vertical erosion on top of the previously calculated 10.1 km. Greater than the previously published estimate of 7.85 km of vertical erosion (Isaacs, 2020), this value remains one of the highest known published estimates for truncation of anticlines, with earlier studies for the Uinta Anticline in Wyoming calculating up to 5.1 km of vertical erosion (Oard and Klevberg, 2008). It must be remembered, however, that this simplified model is an imperfect representation of the fold surface and is thereby only an estimate, as illustrated by the error associated in modeling the limbs of the fold system summarized in Table I. Even so, this approach offers the potential for a consistent mathematically defined methodology that can be translated to sites across regions and compared with similar margins of error and implicit assumptions.

Conclusions

The regional-scale erosion of hundreds to thousands of meters of stratigraphy remain one of the most pivotal yet debated arguments within Flood Geology. Despite the many

Figure 6 ***(left)*****. A simplified diagram depicting Mount St. Helens and underlying fold system of the Lakeview Peak Anticline and Pole Patch Syncline; arrows denote direction of plunge. The line connecting both Lakeview Peak Anticline and Pole Patch Syncline is a 44.2 km transect representing the distance between axes. The star denotes reference point west of Smith Creek Butte used in Isaacs (2020), while distances between that point and the axes are also shown.**

Figure 7 ***(below)*****. Modeled fold geometry (lower curve) resulting from the boundary-value problem and its vertical translation to the uppermost observed fold surface in the Pole Patch Syncline (upper surface). Vertical line represents average elevation in this area and elevation of $y(0)$. Due to sign convention, $y(0)$ is negative (y-value shown by horizontal line) so that nearest anticline appears to the west (negative x-direction) when facing north.**

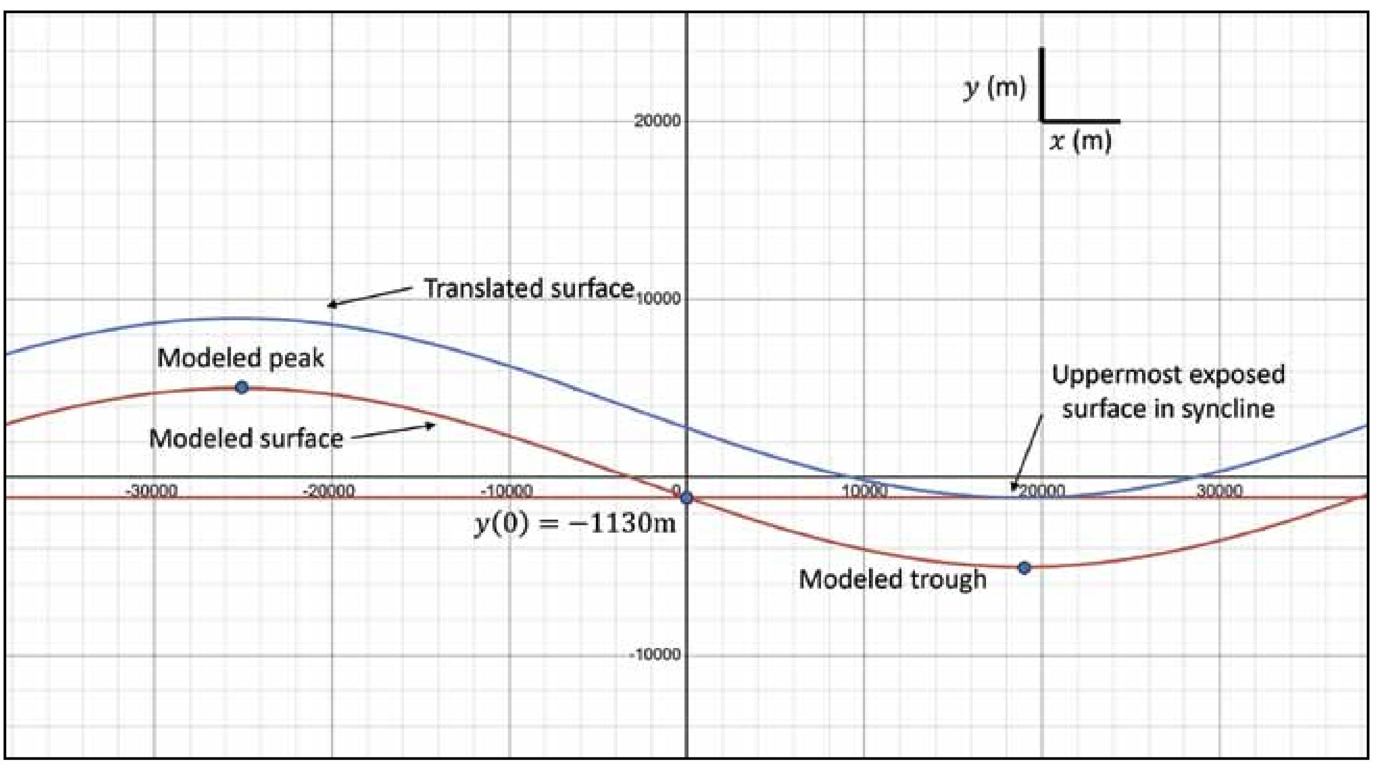

Table I. Comparison of modeled anticline geometry and mapped geometry. Note the percentage error between the model and mapped characteristics of the half-wavelength $\lambda_{1/2}$.

	Mapped	Modeled	Percentage Error
Distance to Anticline	28.0 km	25.2 km	+10.7%
Distance to Syncline	16.2 km	19.0 km	+14.3%
Elevation Difference	NA	6.16 km	NA

previous studies of erosion of anticlines as one illustration of regional-scale erosion, no rigorous mathematical model has been proposed for systematic and repeatable modeling of fold surfaces for erosion estimates. As such, this study investigated differential geometry approaches to reconstruct fold systems.

Using a case study of the Mount St. Helens region, a boundary-value problem was applied to model a symmetric and non-verging fold system. The model performed moderately well (within 15%) in describing characteristics of the half-wavelength of the fold system, giving some level of confidence in the application of this simplified model to this fold system. The modeled surface resulted in a calculation of 6.16 km of vertical relief eroded from the current topography, a value that could be increased to 10.1 km when transferring the modeled surface to the uppermost observed fold surface. Site-specific geology suggests an additional 1 to 4 km of stratigraphy may have rested atop this modeled surface but remains beyond the purview of the mathematical technique.

This application of boundary value problems represents a promising technique to systematically reconstruct fold systems for erosion estimates. The approach requires minimal inputs that are easily acquired from geologic maps (initial elevation and slope, and distance between fold maxima and minima). However, due to the minimal data required to constrain the model, researchers must be aware of the assumptions implicit in the model (e.g., fold system is not strongly verging). Future research may develop alternative approaches using differential geometry that better reflect local site conditions. Even so, this technique represents a first step towards developing an easily deployable yet rigorous approach to create repeatable and consistent erosion estimates.

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