other. The application of Roche's limit to an icy body is then valid to the same extent that it is equally easy to tear apart "chunks" of ice and water.

Stated in another way, in Roche's formulation, the critical distance is that at which the gravitational pull of the larger body just overcomes internal gravitational forces of the smaller body. Once these forces are cancelled, dissolution is immediate and automatic. If however, the internal gravitational forces of a solid planet were cancelled out, it would still be held together by its own tensile forces, which are generally thousands of times stronger than the gravitational forces tending to hold the planet together.

It might seem that a liquid body could not exist in the vacuum of space, but it is known that, if a liquid body is large enough, its cohesive gravitational forces can balance the outward pressure gradient.

Answering the question of whether or not Roche's limit should depend on the size of a liquid body, we note that unless there is adequate mass there never will be a body in the first place, and that all the larger body has to do is to reduce the smaller one to "chunks" of less than the minimum size in order to effect its complete dissolution.

Summary

In summary, it might be stated that, if Mercury or some other deep space invader with an icy satellite had swept close enough to earth, earth's gravitation might indeed have snatched the ice away, but that it would not have fragmented. It might have gone either into stable orbit around the earth, or else into a decaying orbit, which would then have caused it to plunge to the earth, probably having fragmented somewhat due to the effect of air friction on the outside and thermal stresses between the outside and inside.

It might be noted in closing that Patten also invokes his "intruder" as the mechanism of mountain formation on the earth. This viewpoint has a subtle, inherent pitfall. It is now widely accepted by both creationists and evolutionists that the Americas were once joined to Africa and Europe. Many creationists think that Gen. 10:25 is the place where the Bible records the dividing of the original land mass. It should be noted that this was after the flood.

Thus, if the mountains were formed during the Flood even, as Patten supposes, the land masses would not have had the same orientation as they do now. Thus the arcuate form of the mountain chains to which Patten attaches so much significance[4] would not be really significant at all. The land masses might have moved into their present positions after the mountain formation, and thus the present position could not be used to make any inferences about their formation.

References

[1]Patten Donald W. 1988. The Biblical flood and the ice epoch. Pacific Meridian Publishing Co., Seattle, Washington.
[2]Jeans, J. H. 1928. Astronomy and cosmology. Cambridge University Press.
[3]Roche, E. 1850. *Academie de Montpellier (Sciences),* 1:243.
[4]Patten, *Op. cit.,* p. 75ff.

THE RELEVANCY OF ROCHE'S LIMIT TO THE FLOOD-ICE DUMP THEORY

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The flood-ice dump theory of Patten is examined in view of objections to the use of Roche's fragmentation limit. A modified Roche's limit for rigid bodies with tensile strength is calculated and the tensile strength is found to be unimportant for an icy body of appropriate size.

One of the key axioms of the flood-ice dump theory of Patten[1] is the fragmentation of an icy body at a certain distance above Earth known as Roche's limit.[2] The theory suggests that after the demise of the icy visitor, the fragments interacted with the solar radiation and the geomagnetic field; were deflected along the magnetic lines, and descended over the magnetic poles in a period of a few weeks. The arrival of such a large quantity of ice in the magnetic polar regions caused the great ice age.

The strength of any theory depends on its ability to withstand legitimate objections that may arise regarding its claims. One such objection to Patten's theory is the application of Roche's limit (an idealized construction) to a somewhat less than ideal situation. This work seeks to examine several facets of this basic objection and to present mathematical evidence for the fragmentation of an icy visitor near Earth.

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Gravitational Tension Forces

Although the mathematical derivation of Roche's limit (as recorded by Jeans[3]) is somewhat involved, the basic physical idea behind it is relatively simple and in fact is exactly analogous to the lunar tide effect. Due to the inverse square law for gravitational forces, the part of Earth nearest the moon is attracted by a force 7% greater than the part farthest away. The

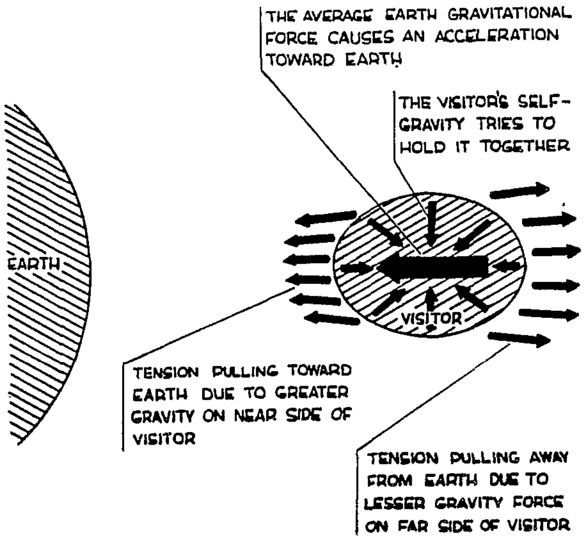

Figure 1. Forces on a visitor to Earth's gravitational

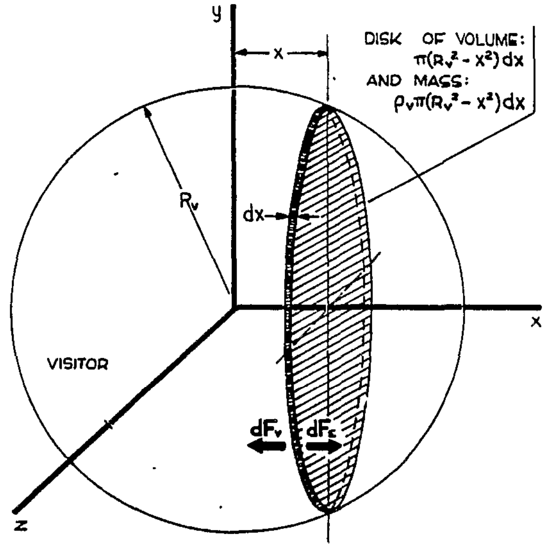

Figure 2. Geometry of a thin disk in a sphere subject to gravitational tension.

average force causes Earth to accelerate toward the Moon, which it does by virtue of its orbit about the Earth-Moon center of mass,

The 7% difference in the force serves to distort Earth, putting it in tension along a line through its and the Moon's centers. Earth's own self-gravitational force acts to hold it together in a near spherical shape.

Applying the analogy, an icy visitor experiences the same tension force due to Earth's gravitational field as does Earth due to the Moon's. The forces acting on such a visitor are shown schematically in Figure 1.

The gravitational tension in a body becomes greater and greater as it approaches Earth. At some critical distance, the tension force caused by Earth will overpower the forces holding the body together, and it will literally be pulled apart. In 1850, Edouard Roche[2] calculated this critical distance for the case of a binary star in circular orbits about the center of mass of the system. He assumed the larger star to be a rigid sphere and the smaller to be liquid.

Searching for equilibrium configurations, he found that if the centers of the stars are closer than 2.44* times the radius of the larger, no equilibrium configurations exist at all. The reason is that the smaller (liquid) star is pulled apart by the gravitational tension from the larger one. The critical distance he calculated has since been labeled "Roche's limit."

*Roche calculated the number 2.44, but this was later refined by Sir George Darwin[4] and found to be 2.4554. It is indeed ironic that Darwin was so familiar with Roche's limit, for it was Darwin who proposed that the Moon was born out of the Pacific Ocean and slowly spiraled out to its present position (tidal evolution theory[5]). His theory is incompatible with Roche's limit because the Moon would fragment if it were so close to Earth.

Refinements on Roche's Model

At first glance it seems somewhat improper to use Roche's simplified model in the flood-ice dump theory. The icy visitor of the theory is not a liquid, nor is it in a circular orbit around Earth. Thus it is appropriate to calculate a "modified Roche's limit" for a rigid spherical visitor with tensile strength.

Consider the configuration of Figure 2. If R_v, the radius of the visitor is somewhat less than r, the distance between the centers of Earth and the visitor, then Earth's gravity field in the visitor is essentially parallel to the x axis. It can be expanded in a Taylor series:

$$F'(x) = -\frac{GM_E}{r^2} + \frac{2GM_E}{r^3}x + \text{small terms}, \quad (1)$$

where F' is Earth's gravity force per unit mass, G is the universal gravitational constant and M_E is the mass of Earth. The first term on the right side of equation (1) is not of interest as it represents the "average force" which accelerates the visitor toward Earth. The second term (gravity gradient term) produces nearly all the tension. Considering only the gravity gradient term, the differential force on the thin disk of the visitor shown in Figure 2 is given by

$$dF_E = +\frac{2\pi GM_E\rho V}{r^3}(R_V^2 - x^2)x\,dx, \quad (2)$$

where r_n is the density of the visitor.

The visitor has its own gravity which pulls the other way on the disk,

$$dF_V = -\frac{\pi GM_V\rho_V}{R_V^3}(R_V^2 - x^2)x\,dx, \quad (3)$$

which was found by adding the x-components of the differential forces on each part of the disk. The net gravitational force on the disk is the sum of the parts due to Earth and the visitor, $dF = dF_E + dF_V$.

$$F(x) = \int_x^{R_V} dF \; ; \qquad (4)$$

i.e., *all* slices beyond x exert a tension force on the surface at x. The average stress on this surface is F(x) divided by the cross sectional area of the surface. The maximum stress will occur at x = O. If this maximum stress exceeds the tensile strength of the visitor, it will be pulled apart. Then the fragmentation distance, $r = r_f$, is determined by the condition; $\sigma_{max} = F(O)/\pi R_V^2$, where σ_{max} is the maximum tensile stress of the visitor. Evaluating the integrals and solving for r_f gives

$$\frac{r_f}{R_E} = \left(\frac{2\rho_E/\rho_V}{1+\alpha} \right)^{1/3} , \qquad (5)$$

where a is the tensile strength parameter,

$$\alpha = \frac{3}{\pi}\, \sigma_{max}/G R_V^2 \rho^2 . \qquad (6)$$

As a point of reference; for a spherical body without tensile strength and of the same density as Earth, this calculation gives $r_f/R_E = 1.256$, which is less than Darwin's 2.4554. There are two reasons why Roche's fragmentation occurs farther out. His star, being a liquid, elongates along a line passing through its and the other star's centers. This elongation produces a greater tension in the liquid star than if it were spherical.

The other reason is that the liquid star is rotating as well as revolving about its partner. The rotation produces a tension exactly analogous to the tension in a spinning flywheel. These two additional tensions cause it to fragment earlier.

Objections to the Fragmentation of an Icy Visitor

Now that Roche's limit has been appropriately refined, the objections to the fragmentation phase of the flood-ice dump theory can be examined. There are basically four objections to the use of Roche's limit as applied by Patten. One is that the density of an icy visitor is not the same as the density of Earth. But applying the correct densities to equation (5) indicates that fragmentation will actually occur at $r_f/R_E = 2.30$, which is 33% farther out than if the densities were the same.

The second objection is that the interaction of two bodies (Earth plus visitor) is an oversimplification, and that the effects of the Moon, Sun, and planets invalidates the result. The response to this is that the gravitational fields of the Moon, etc., also have gradients and also produce a tension (though much smaller) in the visitor. Any additional tension will only cause the visitor to fragment *sooner* than the model of equation (5) predicts. The model is conservative.

The third objection is that the body may not be spherical. Actually, a nonspherical body will experience a moment tending to align its longest axis along a line through its and Earth's centers. This will again cause it to fragment sooner than predicted. Roche's liquid body elongates into this configuration which contributes to its earlier fragmentation.

The fourth objection states that the tensile strength of the visitor will far surpass any gravitational tension force, and the example of the human body (which obviously doesn't fragment) can be cited. The response to this is to apply to equation (6) the tensile strength of ice.

Unfortunately, a strength of materials table won't list such an unlikely building material as ice. A conservative substitute, cast aluminum, is used for σ_{max}. Then for a visitor of radius 250 km (roughly the volume of the ice dump[6]), the parameter a is 1.73 and the fragmentation distance is reduced by 39%. Using the actual strength of ice, it would scarcely be reduced at all.

The physical reason for the small effect of tensile strength is the nature of the gravitational tension forces. The analogy of a "tug of war" game is instructive. If there is only one man on each team, the rope will not break. But if a large enough number are on each team, the rope will break where the tension is greatest (in the middle) due to the cumulative force of all the men. In the same way, the gravitational tension exerted on a small body (such as a human body) is not very significant. But the cumulative gravitational tension in a large body may be very great.

Thus it is seen that in each case, the general trends predicted by Roche's calculation are carried over into the more refined calculation. A rigid icy body of the appropriate dimensions will fragment at a distance of about 2.30 Earth radii from Earth's center.

* * *

It is hoped that this paper is a satisfactory response to Dr. Hoff's questions. Regarding his comment on continental drift, in the author's opinion, this theory is hardly a foregone conclusion—but that is another study in itself.

References

[1]Patten, D. W. 1986. The Biblical flood and the ice epoch. Pacific Meridian Publishing Co., Seattle, Chapter VII.
[2]Roche, E. 1850. *Academie de Montpellier (Sciences)*, Vol. I, p. 243f.
[3]Jeans, Sir J. H. 1928. Astronomy and cosmogony. Cambridge University Press, London, pp. 217-224.
[4]Darwin, Sir G. 1910. Collected works, Vol. III, p. 436f.
[5]*Ibid.*, p. 442.
[6]Patten, *Op. cit.*, pp. 114-115.