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[25]Kaye, Clifford A. *Op. cit.*, p. 43.

[26]Construction records were kindly supplied by Mr. Bill Rudy, Assistant Bridge Engineer, Bridge Office, Division 7 Headquarters, Ohio Department of Transportation, Sidney, Ohio, in a telephone conversation on May 21, 1976. Valuable assistance was also furnished by Mr. Ed Wittle, Bridge Engineer, Clark County Engineers Office, Springfield, Ohio.

[27]Keithley, Willis E., *Op. cit.*

[28]Williams, E. L. *et al.*, 1976. *Op. cit.*, p. 212.

[29]Barr, Thomas C., Jr. 1955. Geological report (in) The caves beyond, Joe Lawrence, Jr. and Roger W. Brucker, Funk and Wagnalls Co., N.Y., pp. 266-273.

[30]Allison, Vernon C., *Op. cit.*, p. 118.

[31]Harris, Robert, *Op. cit.*

[32]Allison, Vernon C., *Op. cit.*, p. 111.

[33]The Olentangy Indian caverns have been formed in the Delaware Blue Limestone which is overlain by the Columbus White Limestone. Cavern tours include the first three levels below the surface (55, 75, and 105 ft.), although the cave is thought to extend as deep as 500 feet. Cave temperature is a constant 54 °F (12.2 °C) and complete air exchange occurs every 25-30 minutes. Only a few small stalactites are observable. Rapid deposition of calcium carbonate in this cavern was brought to our attention by Mr. Randy Helmick, Assistant Plant Chemist, Quality Assurance Department, Foundry Division, Ashland Chemical Company, Cleveland, Ohio.

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THE ROTATION-CURVE OF THE VIRGO CLUSTER OF GALAXIES

GERARDUS D. BOUW*

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The rotation-curve for the Virgo cluster of galaxies and the so-called Southern extension is presented. The two appear distinct in their radial velocity distributions and, in addition, there appears to be another grouping associated with NGC 4261. The masses of the two clusters are $(1.3 \pm 0.2) \times 10^{15}$ *and* $(1.6 \pm 0.2) \times 10^{14}$ *solar masses respectively. The central densities are* $(3.5 \pm 1.0) \times 10^{-25}$ *and* $(2.6 \pm 0.5) \times 10^{-26}$ *gm cm*$^{-3}$ *respectively. Boundary conditions yield an estimate for the intercluster medium of the supercluster of* $(2.2 \pm 0.8) \times 10^{-29}$ *gm cm*$^{-3}$*. The period of revolution of the two clusters about each other is about* 3.4×10^{11} *years; more than ten Hubble ages. This latter factor and the discovery of a previously unsuspected shell wherein the number of direct and retrograde moving galaxies are equal provide further damaging evidence against the prevailing modern cosmogony.*

Introduction

The study of rotation-curves is fundamental to galactic dynamics. This report presents a study of evidence that clusters of galaxies are rotating and that they are stable on "time scales" some ten times greater than the presently held age of the cosmos.

Everyone is familiar with the idea of putting men and satellites into orbit about the earth, moon or other planets. Such behavior is held possible because the gravitational force can be balanced by the centrifugal force. The former tends to draw bodies together while the latter acts in such a way as to draw them apart. In the same way the planets revolve about the sun. It is also observed that stars may go around each other, as is the case for double stars.

The stars, in turn, are organized into larger bodies called galaxies which also appear to be held together gravitationally in the same way as is true for the solar system. Galaxies, in turn, can also be double or multiple and can also be grouped into ensembles called galaxy clusters. Evidence is presented here to show that these clusters are also held together by gravitation, and that they, too, rotate as a whole in just the same manner as the solar system.

Now the gravitational attraction of bodies depends upon their masses and hence it is possible, for example, to deduce something about the mass of the sun from the motion of the planets. Similarly, something can be found out about the masses of double stars; likewise for galaxies and, by extension of the idea, the clusters of galaxies.

Of course, the motion of galaxies cannot be observed directly as that of the planets can. It can, however, be deduced from the Doppler effect, the phenomenon which lowers the pitch of a passing automobile horn. Instead of sound, of course, in the case of stars and galaxies one is dealing with a shift in the color of the object's light as it moves toward or away from the observer.

Simply stated, a rotation-curve is a plot of the rotational velocity against the central distance. To arrive at a rotation-curve for the solar system, for example, one would plot the orbital speed against the distance from the sun. This approach is not very practical nor necessary for the solar system where there are only a few objects which can easily be dealt with separately; but in the case of a galaxy with some 10^{11} objects it is quite practical.

The same is true for galaxy clusters, although they contain only a few hundred member galaxies. Part of the reason is that at present an observer can only estimate the motions of a few stars and no galaxies perpendicular to the line of sight (i.e. in what direction

*Gerardus D. Bouw, Ph.D., receives mail at 43 Kernwood Drive, Rochester, N.Y. 14624.

and how fast in the plane of the sky). To date, only rotation-curves for galaxies have been attempted; and about 100 have appeared in the literature.

There are three basic properties which can be deduced from a rotation-curve. These are: (1) the density as a function of radial distance, $\rho(r)$; (2) the total mass interior to a distance r, $M(r)$; and (3) the surface mass distribution, which will not be discussed here.

Certain assumptions are necessary. These include circular motion of the velocity average in a given locale (i.e. local centroid), and some assumptions about the over-all shape of the object (a spheroid). Newtonian gravitational formalism is also assumed.

Historical Development

To date rotation-curves have only been attempted for galaxies. The first was done by Fritz Zwicky[1] in 1933 and since then there has been a gradual refinement of technique and theory. In 1942 Chandrasekhar[2] derived an analytic expression for the rotation-curve of the Galaxy; but it has not proven to be general enough to fit all galaxies. About that time[3] problems appeared with negative densities at large distance from the dynamic center and to date no satisfactory resolution of that phenomenon has been proposed. Later models have managed to circumvent the negative densities; but there seems to be a certain point beyond which the mass interior to r starts to decrease as r increases. More on this later in the section on mass.

The development of the most popular model to date started in 1959 in a paper[4] which noted that the square of the circular velocity, $v^2(R)$, at a radial distance R is related to the density, $\rho(r)$, according to the relation ship:

$$v^2(R) = 4\pi G(1-\epsilon^2)^{1/2} \int_0^R \frac{\rho(r)\, r^2\, dr}{[R^2 - \epsilon^2 r^2]^{1/2}} \qquad (1)$$

Here G is the gravitational constant and ϵ is the eccentricity of the elliptical cross-section of a sequence of spheroids which are concentric, coaxial, of equal eccentricity, and each of which can be viewed as being of uniform but unequal density (i.e. a spheroidal homeoid sequence). Hence R is the semi-major axis of the outermost shell.

Subsequently[5] it was shown that if $\epsilon = 1$ (actually $\epsilon \geqslant 0.86$ for the two clusters under consideration) then the total mass, $M(R)$, interior to R is given by:

$$M(R) = \frac{2}{\pi G} \int_0^R \frac{v^2(r)\, r\, dr}{(R^2 - r^2)^{1/2}} \qquad (2)$$

The general expression which was found to fit galactic rotation-curves was discovered, in a series of papers,[6-9] to be of the form:

$$v(r) = \frac{v_m r}{R_m} \left[\frac{3}{1 + 2\left(\frac{r}{R_m}\right)^n}\right]^{3/2n} \qquad (3)$$

where v_m is the maximum velocity of the rotation-curve which occurs at a radial distance R_m and n is a constant which is determined by a best fit of the equation to the data. A number of curves of the form of Equation (3) can be summed as their square (i.e. add their kinetic energy contribution per unit mass) so that an even more general form results which can briefly be written as:

Symbols Used in the Text

b — the angular distance perpendicular to l.
b_c — the value of b of a cluster center.
G — gravitational constant.
i, j — integers.
k — the number of velocity curve components summed to make one rotation curve.
K — a constant.
l — angular distance measured from celestial equator along major axis of cluster.
l_c — the value of l of a cluster center.
$m.e.$ — mean error of sample.
M_M — mass of main cluster.
M_0 — total mass interior to the first maximum of the k-components of the velocity curve sum.
$M(r)$ — mass interior to r.
M_S — mass of Southern extension cluster.
M_T — total mass of a cluster.
M_v — virial theorem mass estimate.
n — a constant.
n_i — the value of n for the i^{th} component of velocity curve sum.
N — number of objects in a sample.
P — period.
r — radial distance from center of mass.
R — radius of outermost spheroid under consideration.
$\bar{r}$ — average value of r of sample.
R_m — the radial distance at which v_m occurs.
R_{m_i} — the value of R_m for the i^{th} component of velocity curve sum.
R_{MC} — distance from center of main cluster to dynamic center of both clusters.
R_{SC} — distance from center of S. ext. cluster to mutual dynamic center.
$\bar{v}$ — average velocity of sample.
$v_i(r)$ — the i^{th} component in the rotation curve sum.
v_m — maximum value of velocity of rotation curve.
v_{m_i} — the value of v_m for the i^{th} component of rotation curve sum.
v(R) — circular velocity at R.
α_c — right ascension of cluster center.
δ_c — declination of cluster center.
ϵ — eccentricity of the elliptical cross-section of the spheroid.
$\rho(r)$ — density at r.
σ — standard deviation of average velocity.

$$v(r) = \left[\sum_{i=1}^{k} v_i^2(r)\right]^{1/2} \qquad (4)$$

where $v_i(r)$ is the value of Equation (3) for the i^{th} peak at v_{m_i} and R_{m_i}.

The galaxy cluster case, however, differs significantly from the galactic case. First of all, from the relative sharpness of the spectral lines of the latter there does not appear to be a sizeable fraction of stars which deviate significantly from the mean local circular motion. Now this fact may square with the cloud-collapse hypotheses of galaxy formation, but seeing that the local stellar count density in the vicinity of the sun, coupled with radio-based estimates of the interstellar mass-density is not enough to be explainable in terms of a cloud-collapse origin[10] one is not justified in assuming that all the members of a galaxy cluster revolve in the same direction.

In fact, Figure 1 reveals a zone in which the number of galaxies moving in direct orbits equals the number moving in the opposite direction, so that the average velocity ends up being zero. Although the problem of a cloud collapsing to a galaxy is more readily solved than that of a cloud collapsing into stars, this observation of the Virgo cluster's orbital mixture could prove to be every bit as difficult for the cloud-collapse hypotheses to overcome.

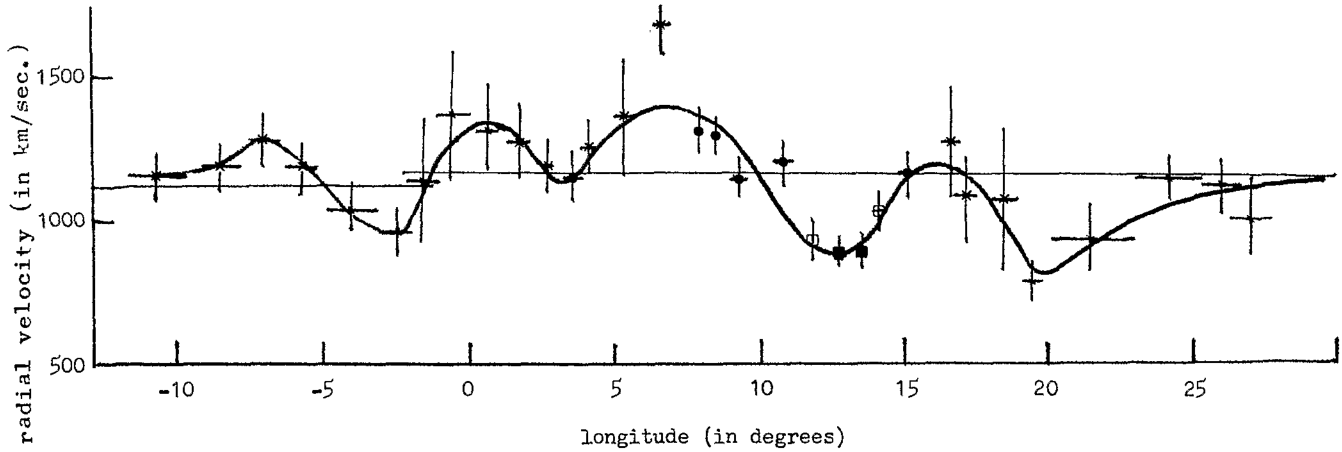

Figure 1. The raw rotation-curve before rectification, showing the radial velocity, in km/sec. (plotted vertically) vs. the longitude in degrees. The number N of objects included in the average for a point is indicated by the point as follows: asterisk, fewer than 5; cross, 5 or more but fewer than 10; dot, 10 or more but fewer than 20; hollow square, 20 or more but fewer than 30; solid square, 30 or more. The upper horizontal line is the radial velocity of the main cluster; the lower is that of the Southern extension. Members of the group NGC 4261 could have been added, at about 7 degrees and extending from about 1700 to 2500; this suggests a rotation-curve. The last four data points at the right end of the main cluster's rotation-curve involve the galaxies proposed as members in this paper. The dips to the mean velocity of the cluster at 3.3 and at 16 degrees are zones in which the number of retrogade objects equals the number of direct; they seem to conform to a 6.5 degree cycle.

As a result of the failure of the unidirectional orbit hypothesis one must look at each member galaxy separately, and if necessary, change the sign of its radial velocity relative to the center of mass in order to have all objects moving in the same sense. For the inner portions of the cluster this amounts to assuming equipartition of velocity (analogous to equipartion of energy in physics). Hence the observed radial velocity average will be half the true value and the tangential velocity average (if it could be observed) would be $\pi/4$ of the true velocity. Thus, the observed average radial velocity should be multiplied by two, yielding the upper curves of Figure 4.

It was found that whereas Equation (3), with v_m = 250 km/sec, $R_m = 3°.15$ times the scale factor (see Figure 4) and $n = 1.75$, does envelop the observed double peak for the main cluster of Figure 1, it does not drop off fast enough to account for the subsequent decline of the curve nor for the decline indicated by the data in Figure 4. A more powerful form of Equations (3) and (4) was searched for and found to be:

$$v(r) = 2r \left\{ \sum_{i=1}^{k} \left[\frac{3}{1 + 2\exp[\{\frac{r}{R_{m_i}}\}^{n_i} - 1]} \right]^{3/n} \left(\frac{v_{m_i}}{R_{m_i}}\right)^2 \right\}^{1/2} \quad (5)$$

where $k = 1$ for the Southern extension's rotation curve in Figure 4b and $k = 3$ for the main cluster's rotation curve in Figure 4a.

Data Reduction

In all 111 galaxies with observed radial velocities are included in this study. Because of their high radial velocities (the highest of all in the sample) and because of their sizeable distance from the dynamic center, NGC's 4593* and 4939 were deleted from cluster membership. Although not previously recognized as cluster members, NGCs 4064, 4494 and 4565 meet all membership requirements and have been included here. The right ascensions and declinations (the celestial analogues of longitude and latitude respectively) of the member galaxies were first reduced to a cluster-coordinate system.

Since the galaxies project to an ellipse on the background sky, the major axis was taken as the equator of the cluster and the resulting line intercepts the celestial equator at 1950 right ascension $12^h41^m \pm 2^m$ and at an inclination of $74°.8 \pm 0°.3$ measured counter-clockwise from the celestial equator. The cluster longitude is then measured clockwise-as seen from the northern hemisphere-along the cluster's equator and measured from the aforementioned point where the cluster's "equator" intercepts the projection of the earth's equator upon the sky.

To determine the center of mass position and velocity a raw rotation-curve was plotted (Figure 1). This was done by the sliding window technique, namely, by pigeon-holing the members into 1-degree intervals of cluster longitude, l, averaging and then taking a weighted average of the result with the averages of the intervals on either side of the given longitude interval. In effect this smears the data to about 3-degree resolution.

Figure 2 is also a smeared curve of the average value of latitude, b, and shows the deviation of the sample from the chosen equatorial plane. It indicates that the cluster is not totally relaxed. The time required for a galaxy to cross the mean amplitude is about 10^9 years (given the velocity dispersion times $\pi/4$).

Figure 3 is the rotation-curve in latitude which has been divided into the left and right halves of Figure 1 along with a composite.

It is apparent from Figure 1 that a division of the cluster into a main body and the Southern extension is justified; the division between the two occurring near $l = -1°.5$. On the basis of the symmetries of the curves

*The NGC designation refers to the number assigned the object in the NEW GENERAL CATALOGUE of Nebulae and Clusters of Stars published by J. L. E. Dreyer in 1888. In 1971 it was reprinted by the Royal Astronomical Society in London.

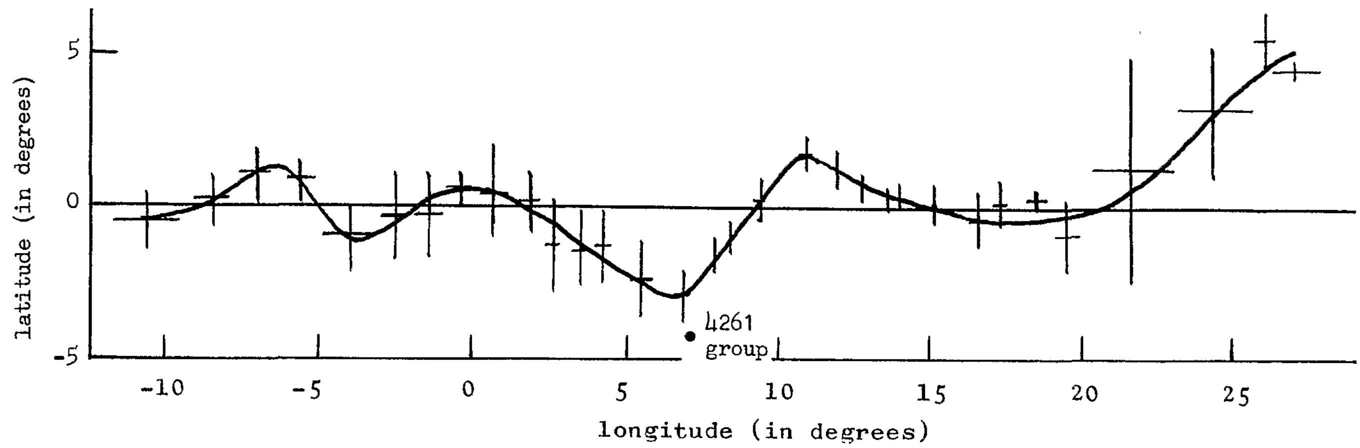

Figure 2. This shows the latitude of objects vs. longitude, both in degrees. Thus it indicates the deviation of the average latitude of the sample from the assumed equatorial plane of the cluster. Again the last four data points on the right, for which *l* is more than 20 degrees, involve the proposed members. If the chosen equatorial plane differed significantly from the true plane then a straight line of constant slope would be expected. Instead, the periodicity of the curve suggests that the cluster is not relaxed. The group NGC 4261 is shown here as a single data point. It could not be the cause of the distortion, for its mass is only about 0.6% that of the cluster, while the total amplitude of the distortion is about a megaparsec.

Table 1.

Parameter	Main cluster	S. Ext. cluster
l_c	10°.2 ± 0°.2	−5°.0 ± 0°.3
b_c	0°.6 ± 0°.4	0°.8 ± 0°.4
$\overline{v}_c$	1160 km/sec	1110 km/sec
α_c	$12^h 33^m$	$12^h 52^m$
δ_c	10° 31′	−05° 00′

Table 3.

	Main cluster (k = 3)			
Parameter	i = 1	i = 2	i = 3	S. Ext. (k = 1)
v_{m_i}(km s^{-1})	660	605	110	330
n_i	0.75	2	2	0.9
R_{m_i}(Mpc)	0.22	1.23	2.64	0.55
r_{m_i}(deg)	1	5.6	12	2.5

Table 2.

N	v	m.e.	r	m.e.	comments
Main Cluster:					
1	1044	22	0.47	0	1° average
8	633	168	1.52	0.10	2° average
27	479	38	2.11	0.11	
43	561	30	2.77	0.13	
47	489	29	3.41	0.12	
34	587	38	4.26	0.13	600 k/s w. 4261 as one
25	442	39	5.36	0.19	535 k/s w. 4261 as one
23	582	37	6.72	0.14	599 k/s w. 4261 as one
21	507	34	7.20	0.15	
16	504	50	8.00	0.22	
9	287	44	9.44	0.26	
8	246	49	10.33	0.31	
6	217	60	11.68	0.92	5° average
1	88	27	16.05	0	1° average
12	87	76	18.31	1.00	S. Ext. cluster
Southern Extension Cluster:					
2	148	36	0.61	0.31	1° average
3	243	98	0.91	0.35	2° average
4	182	92	1.24	0.42	
5	331	35	2.89	0.44	
6	261	43	3.77	0.40	
6	276	46	4.24	0.32	
4	209	64	5.32	0.39	
3	62	45	7.12	1.38	5° average
2	92	57	8.12	1.65	4° average
1	41	26	9.77	0	1° average

of Figures 1 through 3 the center of mass parameters for the two clusters are tabulated in Table 1.

When the rectification described in the previous section using these parameters was accomplished the results were plotted as Figure 4. Again the smearing was done as described above, but not all points are smeared to the same degree. This can be seen in the comments column of Table 2 which lists the relevant averages for the two clusters. All error bars in the figures and columns labelled "m.e." are mean errors. In Table 2 "*N*" indicates the number of galaxies considered in the average, *v* is the average observed radial velocity corrected for retrogrades.

For the Southern extension case the contribution of the rotation-curve of the main cluster has been removed. The column labelled *r* lists the average distance from the center of mass of the *N* objects. The members of the NGC 4261 group (NGC's 4260, 4261, 4270, 4273 and 4281) were not included in the data of Table 2 but they are treated as one object in three entries in the "Comments" column. Their deletion does not appear to have a significant effect on the final results.

The Density Distribution

Given that the mass is expressed by combining Equations (2) and (5) then, from the definition of a spheroidal homeoid sequence, it follows that the density can be expressed as:

$$\rho(R) = \frac{3M(R)}{4\pi R^3 (1-\epsilon^2)^{1/2}} \qquad (6)$$

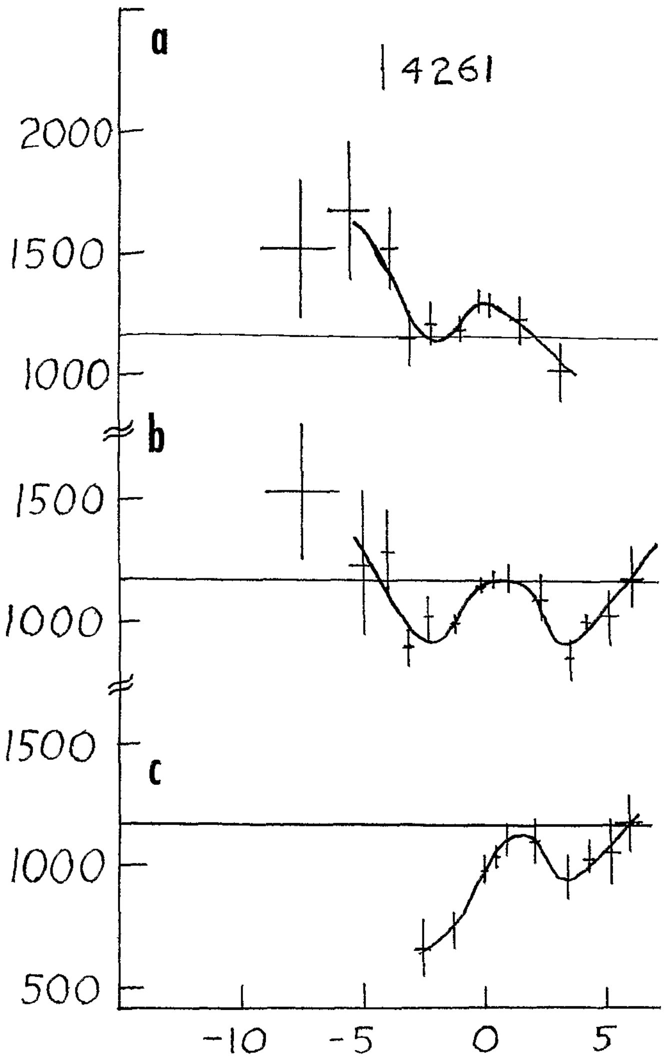

Figure 3. The rotation-curve along l = 10.2 degrees, or in latitude. Again radial velocity is plotted vs. average latitude; note, though, that the vertical scale starts over again at intervals. Part a includes all objects having l no more than 10.2 degrees; part c shows that rotation curve for the rest of the cluster, i.e., for l more than 10.2 degrees. Part b shows the data combined, i.e., all l, to show the entire cluster's rotation-curve in latitude. The horizontal lines correspond to the radial velocity of the center of mass as defined by the symmetry of Figure 1.

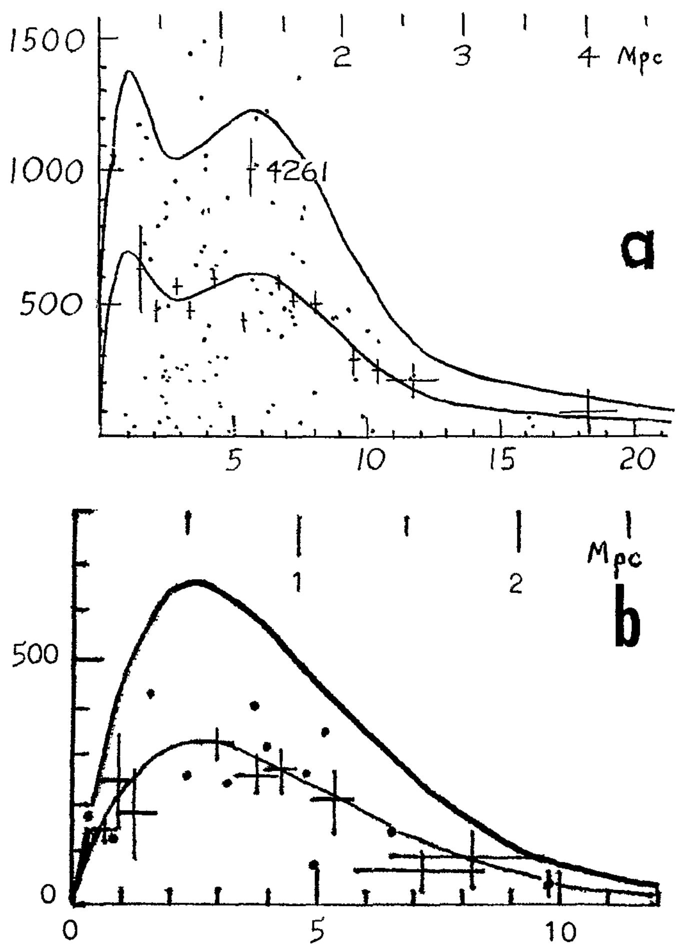

Figure 4. The rectified rotation-curves. All of the data points are plotted. Part a (top) is for the main cluster; b (bottom) is for the Southern extension, contribution of the rotation-curve for the main cluster having been subtracted out in accordance with the formalism of Equation (4). As expected, the upper ($2\bar{v}$) curves envelope nearly all of the data points, and appear to be on the high side beyond about 8 degrees where the postulate of equipartition of velocity breaks down. In both parts the vertical scale shows radial velocity; the lower horizontal scale of each part shows distance from the center of mass in degrees, the upper horizontal scales the same distance in megaparsecs. This scale is based on an assumed distance modulus of 30.5, which corresponds to a distance of 12.6 megaparsecs. Both the NGC 4261 group and the Southern extension cluster are presented as an averaged unit in part a.

The result for each cluster is presented in Figure 5 and is based on the values of the parameters to Equation (5) tabulated in Table 3.

The central density for the main cluster's rotation curve is about the mean density of a galaxy, namely 1.18×10^{-24}gm cm^{-3}; but this may be rather on the high side since the curve $i = 1$ is open to challenge. If it is assumed that the density curve of the main cluster should run about parallel to that of the Southern extension then a central density of about $(3.5 \pm 1) \times 10^{-25}$gm cm^{-3} is indicated. The central density for the Southern extension is $2.6 \pm 0.5 \times 10^{-26}$gm cm^{-3}. Both these values are at least about two orders of magnitude greater than the previous estimate of 2.9×10^{-28}gm cm^{-3} based on the observed light.[11] This result is consistent with the general discripancy that exists between dynamically determined masses and mass-to-light ratio determined mass estimates for galaxy clusters. It is hoped to investigate this further in a subsequent paper.

By defining edges to the clusters (at 25° for the main cluster and 12° for the Southern extension) one arrives at an estimate for the density between clusters belonging to the supercluster of $(2.2 \pm 0.8) \times 10^{-29}$ gm cm^{-3} which is about two orders of magnitude greater than the light-determined value of 3.1×10^{-31}gm cm^{-3} estimated previously.[12]

Total Masses

As was mentioned in the historical development section, there is a problem with mass-reversal. It has been shown[13] that this follows from the theoretical rotation curve and that the mass has an inverse dependence upon the radial distance, R, for large values of R. For the galactic case this reversal is rarely, if ever, reached

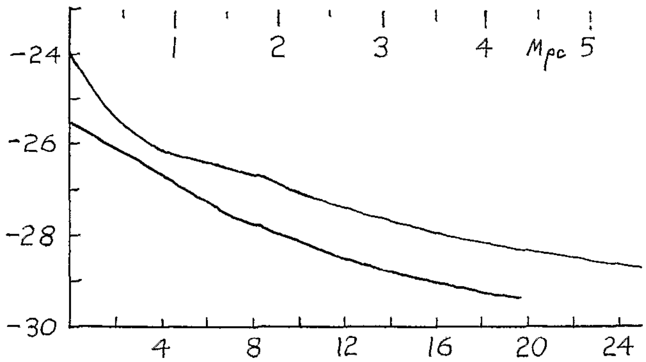

Figure 5. The density. The vertical scale shows the logarithm to base 10 of the density; the lower horizontal scale, the distance in degrees from the dynamic center; the upper horizontal scale, the same distance in megaparsecs. The upper curve is for the main cluster; the lower, for the Southern extension. The steep rise near the nucleus of the main cluster is due to the i = 1 curve, the first maximum of Figure 4; and it is subject to question.

in the optical body of the galaxy; although it can be reached in the radio body. It can be shown, by equating centripetal and gravitational forces and substituting Equation (5) (with k = 1) for the resulting velocity dependence, that for large values of R the dependence of the mass on R for the cluster case goes as:

$$M(R) \propto \frac{R^3}{e^{\frac{3R^n}{n}}} \tag{7}$$

To circumvent the mass reversal it was assumed that the mass evaluated by combing Equations (2) and (5) was valid up to the point R_m, and has a value M_o. Subsequently, the rest of the cluster was divided into shells of uniform density each, the jth of which has a mass M_j, so that the total mass interior to a distance R from the center of mass is given by the sum of M_o plus the contribution of all m shells exterior to M_o:

$$M_T = M_o + \frac{4}{3}\pi(1 - \epsilon^2)^{1/2} \sum_{j=1}^{m} (R_j^3 - R_{j-1}^3)\rho_j \tag{8}$$

The resulting masses are:

$$\begin{aligned} M_T(\text{main cl.}) &= (2.6 \pm 0.4) \times 10^{48}\,\text{gm} \\ &= (1.3 \pm 0.2) \times 10^{15}\,\text{solar masses;} \\ M_T(\text{S, Ext.}) &= (3.3 \pm 0.4) \times 10^{47}\,\text{gm} \\ &= (1.6 \pm 0.2) \times 10^{14}\,\text{solar masses.} \end{aligned} \tag{9}$$

The error terms are based on several different starting values of M_o.

If the assumed physics is consistent over the scale of the cluster then one expects that the distance from the center of mass of the main cluster to the center of mass of the double system, R_{MC}, and the distance from the center of mass of the Southern extension to the combined center of mass, R_{SC}, be related to the ratio of the masses of the two systems according to the expression:

$$\frac{R_{MC}}{R_{SC}} = \frac{M_S}{M_M} \tag{10}$$

where M_M is the mass of the main cluster and M_S is the mass of the Southern extension cluster. Now the radio center of the main cluster, VirA, is located at about R_{mc} = 2°.8 and if it is assumed that that is also the dynamic center of the main cluster then, according to the data of Table 1, the mass ratio should be 0.18 ± 0.04. Assuming, on the other hand, that all galaxies in the main cluster have equal masses, then its center of mass occurs at R_{mc} = 0°.9; corresponding to a mass ratio of 0.06. VirA does not occupy a central position when the cluster's symmetry is taken into account. Neither are the galaxy masses independent of position in the cluster; hence, averaging extremes, the mass ratio is 0.13 ± 0.07. The clusters are 15°.2 apart which is the maximum size which either cluster can be assumed to have in this case. This yields a mass ratio of 0.15 ± 0.02; within the above range.

Refinement of the Virial Theorem

The virial theorem was originally developed in the kinetic theory of gases to express balancing of the motion of molecules against their mutual attraction. Here, instead of the molecules are galaxies; and gravity is the mutually attractive force. In the latter case the usual form is:

$$M_v = \frac{3R\sigma^2}{KG} \tag{11}$$

where R is usually taken to be the cluster radius, σ is the standard deviation of the velocity dispersion and K is taken to be a constant which is a weak function of cluster parameters and galaxy size. Estimates for K range from 0.29 to 3; the former being based on the assumption that the cluster can be approximated by a polytrope of index five.[14]

Since there are two rotation-curves, one can use the resulting mass determinations to determine an empirical value for K. Doing so and dividing the result into the factor of three in Equation (11) yields:

$$M_v = (3.14 {}^{+1.15}_{-0.68}) \frac{R\sigma^2}{G} \tag{12}$$

The error range in the above expression is rather large primarily because of the difference between the two clusters. There is some reason to assume that a larger sample from the Southern extension would increase the velocity dispersion enough to bring it more in line with that of the main cluster, thus decreasing the error range.

Period of Revolution

It is apparent from the data that the galaxies considered here can be viewed as members of a double system. The period of revolution, P, can be expressed as:

$$P = \frac{2\pi R}{V} \tag{13}$$

which is a continuous function of R. By considering the center of mass of the Southern extension cluster as the outermost member of the main cluster one finds that the revolutionary period of the system (velocity equipartition not assumed) is 4.1×10^{11} years if the data in Table 1 is used or 2.7×10^{11} years if Figure 4 is used. These estimates are some 10 to 35 times the presently held age of the cosmos. One could ask whether or not such a period is compatible with the violent origin of the big-bang model.

Conclusion

Not surprisingly the mass determination based on the rotation-curve is compatible with that of the virial theorem. Another method capable of yielding a mass estimate involves counting the number of galaxies in the cluster with some estimate for their individual masses based on their total light output. When this is done the resulting mass is anywhere from about a tenth to a hundredth or more of the virial theorem mass estimate.

Discussion of the various resolutions proposed for that mass discrepancy will be reserved for a future article. In connection with the mass discrepancy, the application of the virial theorem to clusters of galaxies has been challenged. But that application is justified by the rotation-curve's mass estimate.

Relevance to Creationism

In the past, the mass anomaly has been used as an argument for a young cosmos. This has been done on the basis that according to the velocity dispersion (virial theorem) galaxy clusters should have disipated billions of years ago, given the "count" mass estimates.[15] Valid though such an approach may be, it appears here that these clusters can be viewed as bound systems exhibiting, as indicative of their "boundedness", the property of rotation. Hence, such an approach may not be sound in light of this evidence, or may need to be reconsidered. This approach still appears to be applicable to those cases where doubles or chains of galaxies have extremely discordant radial velocities.

Yet this does not give support to the evolutionist's stance; for even though one argument is thus removed, two further arguments remain. As noted before, Figure 1 reveals two "dips" in the raw rotation curve of the main cluster. One of these occurs at $l = 3°.3$ and the other at $l = 16°.2$. These are not zones where objects are "standing still" with respect to the center of mass as one might at first suspect. If they were then there would be a clustering of points toward $v = 0$ near $R = 6°.5$ in Figure 4a. Instead it appears that there is a shell of radius $6°.5$ (1.4 Mpc.) where the number of directly revolving galaxies equals the number revolving in a retrograde sense.

Inspection of Figure 3b shows the same effect in the latitudinal rotation curve with the suggestion that the prevailing direction of rotation may actually reverse beyond that distance. In this case, however, the effect appears about a degree closer to the center of mass than in longitude. Hence, one can tentatively conclude that a spheroidal shell, of eccentricity 0.5, exists inside the cluster in which the orbital directions are totally mixed. Interestingly enough, this shell appears to coincide with the ellipse which describes the outline of the cluster at their respective poles and yet appears to have about half the equatorial radius of the shape-describing ellipse. The first key point for the creationist stance is that no evolutionary model can explain this phenomenon.

The second point involves the time scale indicated by the very existence of a rotation curve as presented here. According to the usual figures quoted for the age of the universe the outermost members of the clusters have only had time to make about a tenth of a revolution. Since the relaxation time for such a system is at least several revolutions it seems rather strange that the motion, usually assumed to have been random initially, could have sorted itself out to the degree that it has and to the radial extent that it has in the evolutionist's time frame. On the other hand, since the Southern extension cluster has not been tidally disrupted by the main cluster it would appear that the system is not several revolutions old.

Finally, one may invoke the standard design argument here, too. It has been noted that matter appears to be distributed in a hierarchy, ranging from planets, to stars, to galaxies, to galaxy clusters and, apparently beyond to superclusters (i.e. groupings of galaxy clusters). Since all but the latter have been shown to rotate,† adding consistency to the hierarcy, and since an arrangement of things into a hierarchy implies planning and design, does that not point to the creation of these objects as being an organized system from the beginning?

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H. J. MASSINGHAM: CHRISTIAN ECOLOGIST

W. J. Keith*

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As the ecology movement became prominent in the last few years, Christians have sometimes been suspicious of it. However, since the aim of ecology is to make the world a better place in which to live, or to keep it from becoming a worse one, it would seem right for Christians to sympathize, and, if possible, to help.

A good way to show that something is possible is to show that it has been done. Thus, the purpose of this article is to show that it is possible to be a Christian and an ecologist by recounting something about a man who was both.

The life of the English writer and thinker H. J. Massingham (1888-1952) developed into a long, often painful, but ultimately successful quest for rural roots and spiritual fulfillment.

Massingham was born into a free-thinking, Liberal, late-Victorian family which was established in a progressive and thoroughly urban milieu. He ended his life as a firm believer in Christianity, a conservative (in the non-party sense that he advocated the conservation of traditional values) and a country-dweller who had abandoned the city and made his home in a rural environment.

A free-lance writer for most of his working life, Massingham produced numerous books on a wide variety of topics. He first made a name for himself as a nature-writer in the style of W. H. Hudson, but later extended his range to include more general regional and topographical subjects. He was always eager to further the cause of rural craftsmanship and traditional husbandry.

Although Massingham never became a practising Catholic, he was baptised into the Catholic Church in the early 1940s, his motive: "I wanted, so to speak, to sign on".[1] This involvement helps to explain the cluster of books written at this time which were concerned directly with the relation between religious beliefs and practices and the cultivation of the land. The books are his autobiography *Remembrance* (1942), *The English Countryman* (1942) and, perhaps most significant of all, *The Tree of Life* (1943).

One Over-Riding Theme

The over-riding theme of Massingham's 1943 book was best expressed by the extract from a letter which he had received from an unnamed naval lieutenant which he used to open his first chapter.

> I feel that the loss of the love of the land for its own sake and the loss of the Christian religion are the greatest tragedies this country has ever suffered.[2]

For Massingham, these two were inextricably interconnected.

This ecologist came to believe that the "ultimates of life"[3] were represented in the sacred trinity of God, Man and Earth. Massingham found symbolic physical realization of this in the pattern of the medieval village-community where the open fields clustered around the manor-house and cottages which were all dominated by the hallowed fabric of the village-church.[4]

For Massingham, this analogy was no accidental parallel, for both are foreshadowed by, and implicit in, the pattern revealed in the Gospels: "The triune relationship of the good earth, the good husbandman and heaven over all is truly contained in the life of Christ" (*TL*, 26).

Massingham's Christ was, first and foremost, "the Christ of the Trades." "The King of Kings," he insisted, "was born in the village cow-byre" (*TL*, 18). Christ's mother was a peasant, Joseph a carpenter, and homage was paid to him at his birth by unlettered shepherds. Christ was born into a rural area ("The eternal 'I Am' made his temporary home with the most immemorial of all human settlers on the cultivated earth" [*TL*, 20-1]). He taught through parables drawn from farming and husbandry and instituted "the informal ceremony of the Last Supper, wherein the unity between nature and the new faith is expressed in the sacramental aspect of the bread and the wheat" (*TL*, 25). The relation between his life and teachings and the eternal processes of country life was both natural and organic: "If the birth of Christ be the meeting of man and God, the farmyard is the meeting-place of man with nature" (*TL*, 18).

Massingham stressed the rural matrix of Christianity because he was aware that the temptation to stress the spiritual world at the expense of the physical creation

*W. J. Keith, Ph.D., is with the English Section, University College, University of Toronto, Toronto, Ontario, Canada.