Information Theory, Consequence Operators, and the Origin of Life

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Abstract

In this article, the Gitt (1997) concept of information as it is represented by a mental-like sequence of activities is compared with the mental activity represented by consequence operators. It is shown how consequence operators model mathematically these Gitt notions and how a specific ultralogic and four ultrawords yield an identical foundation for Gitt's information theory scenario for the origin of life. Consequence operator theory, as a model for Gitt information theory, is used to establish that, relative to this model and without external modification to the processes, it is not possible, using fixed pragmatic information, to increase or decrease the complexity of a biological entity by selecting from two distinct independently produced biological entities as these entities are characterized at the apobetic level. This signifies that if random *mutations* are *random* alterations in the characteristics of a biological entity that are, necessarily, not dependent upon the original biological characteristics, then this model would tend to disallow random mutations as a viable source for biological alterations. On the other hand, for fixed pragmatic information, an increase or decrease in complexity by selecting from two distinct apobetic level biological entities is possible if a very special dependency exists between their characteristics. Further, complexity can also be increased by applying the semantic level consequence operator to an increased portion of the information contained within the genetic code in the DNA, information that exists originally.

Introduction

Information theory as introduced by Shannon (Shannon and Weaver, 1964) is often understood to be a statistical approach to the formation of a language from strings of symbols. However, many researchers (Gitt, 1997; Steinbuch, 1968; von Weizsäcker, 1974) state that this approach to information theory does not capture its actual essence. Indeed, relative to the statistical approach Weaver (Gitt, 1997, p. 45) is quoted as writing: "Two messages, one which is heavily loaded with meaning and the other which is pure nonsense, can be exactly equivalent . . . as regards information." Rather than being defined in terms of statistics and combinatorial mathematics, can the actual *mental activity* nature of information be discussed successfully?

Werner Gitt (1997) has presented to the scientific community a collection of "Theorems" that appear to capture the actual mental activity associated information. Gitt uses a five level approach, where the information content of the statistical level is extended significantly. This approach is utilized to model the origin of life question as well as to model a series of scriptural statements. Gitt's thesis is that it is mental-like behavior that is the true essence of information theory. Is this Gitt approach equivalent to a another approach the foundations of which have been extensively investigated previously?

In a series of articles (Herrmann, 1982, 1984, 1985, 1986a, 1986b, 1988, 1994a), a newly devised method to model mathematically the origin of and the time development for a *natural system* is discussed in a nontechnical yet detailed manner. (Note: Most italicized terms are either specifically defined within the body of this article or in the glossary.) This model shows that it is rational to assume that the most basic behavior that can be associated with the development of a natural system is a behavior that is being mirrored by processes that resemble those associated with an infinitely powerful mental activity. Further, this modeling process, when interpreted linguistically, is shown to correspond to numerous scriptural statements. Hence, this correspondence verifies the scientific rationally of these scriptural statements.

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The Herrmann modeling process is chosen so that it is compatible with all known forms of scientific deduction. Consequently, the results are all relative to the most basic foundation for the origin and development of a natural system and are not concerned with a particular deductive pattern that one might associate with a particular theory. It was assumed that the reader of these past articles could, using the discussed methods, investigate various well-known scientific *inner-logics* that are associated with various theories and, without difficulty, correlate these inner-logics to the *ultralogics* discussed in these articles. Unfortunately, this assumption has not been fully realized. Consequently, within this article, a specific and somewhat complex illustration is given relative to "information theory" as this theory is presented by Gitt (1997) and as Gitt information theory relates to the origin and existence of biological life. This illustration serves as a prototype for all discussions relative to a specific concept that requires some form of logical deduction or similar mental activity, and that is relative to a specific natural system.

A few technical aspects will be recalled; but, the actual mathematical structure with its in-depth construction is rather technical in character and will not be discussed, in detail, within this article. However, partial or full discussions can be found in Herrmann (1993, 1994b, 1999). Although a few formal definitions will be given, in most cases, what is presented here are notions that are intuitively rather than formally defined. As discussed in Herrmann (1994c), it is not difficult to show that all scientific communication can be represented by finite strings of symbols taken from a finite alphabet $\mathcal{A}$. Using modern digitizing techniques and electronics, such finite strings of symbols also include visual and audio impressions. The set of all such nonempty finite strings of symbols is called a language $\mathcal{L}$. (Note: This is not a language as described by Gitt.) Each nonempty finite string of symbols is called a *word* or *readable sentence*. Among the collection of all words, there are specific ones that correspond to meaningful strings of symbols that appear in a *discipline dictionary*. This discipline dictionary corresponds to Gitt's *language elements*. Further, there are readable sentences that are constructed from members of the discipline dictionary by means of a set of formalized or formalizable rules. (Note: The rules themselves are also readable sentences.) These rules form the *syntax* for a language. For the case of digitized visual and audio impressions, this syntax is the construction of the electronic instrumentation and required software that will translate a specific collection of strings of symbols into the actual electronically presented impressions; i.e. computer or TV screens, and speakers. These syntactically constructed strings of symbols are also members of $\mathcal{L}$ and form what is termed as a *meaningful set of readable sentences* or *meaningful words*, $\mathcal{L}_{\mathrm{M}}$, and various nonempty subsets of $\mathcal{L}_{\mathrm{M}}$ correspond to Gitt's second level of information (1997, p. 57), the *syntax level*.

Equivalence Classes

Prior to discussing the mathematical entity that gives a rather general model for many types of human mental processes, one technical aspect for such modeling needs to be explored. This aspect was assumed to be self-evident in some of the previous Herrmann articles written on this subject. However, this "self-evidence" will now be further detailed. Different natural languages use different discipline dictionaries as well as different syntactic rules. In formal logic, the same symbols and same syntactic rules yield different strings of symbols that are said to be "logically equivalent." Throughout various deductive arguments logically equivalent strings of symbols can be substituted, one for another, and the logical argument remains valid. This type of equivalence is defined in a strict and not controversial manner. Types of equivalence are necessary when natural or native language descriptions are used. However, a definition as to the equivalence of different native language strings of symbols is necessarily more intuitive in character.

First note that to comprehend properly a set of symbol strings that contains more than one string of symbols it is usually required that the strings be presented in a specific order. When necessary, the use of a spacing and punctuation symbols would allow any finite set of such symbol strings to be considered as a set that contains but one string of symbols. However, even under a single symbol string notion, various types of equivalence are required for natural language descriptions. Intuitively, this equivalence is captured within the statement that "two strings of symbols are *saying the same thing* or have the *same relative meaning*." It is necessary that a form of mental activity and general consensus be considered in order to come to this conclusion. Whatever technique is used for this purpose, such a concept appears to have universal acceptance or else it would not be possible to "translate" efficiently from one native language to another and ever hope to achieve the same intended results being communicated by two different strings of symbols. Of course, the same idea of equivalent symbol strings would hold within a specific meaningful language as well. The equivalence of two different strings of symbols probably depends upon the term "thing" presented above. But, in any case, such an equivalence would need to be accepted by the vast majority. In some cases, "testing" can be used to determine whether different symbol strings yield equivalent mental results; that they are, indeed, "saying the same thing."

The set of all "equivalent" strings of symbols taken from $\mathcal{L}_M$ and that have been adjudged to be "saying the same thing" is called an *equivalence class*. Distinct equivalence classes are disjoint, that is they have no members in common, and every member of $\mathcal{L}_M$ is contained in some equivalence class. The self-evident notion previously made but not explicitly stated is that the actual set $\mathcal{L}_M$ that determines the domain and codomain for our consequence operators is a set composed of one and only one member, a *representative*, from each of these equivalence classes. In all that follows in this article, the exact same symbol string representation for an equivalence class will be used. This procedure is tacitly assumed.

Consequence Operators

Although our next object of significance is defined on the set of all subsets of $\mathcal{L}$, its application is usually restricted to the set of all subsets of some nonempty $\mathcal{L}_1 \subset \mathcal{L}_M$ (Herrmann, 1993, pp. 1 and 24). Using a weak set-theory, Tarski (1930) models a fundamental essence for mental activity and uses this model to investigate deductive logic. It was discovered in 1970 (Wójcicki, 1970), that a slight generalization of the Tarski ideas leads to significant additional algebraic properties. A *consequence operator* is a function Cn, in the general sense, that is applied to each $A \subset \mathcal{L}_1$ and gives $Cn(A) \subset \mathcal{L}_1$ and which satisfies three axioms. (1) $A \subset Cn(A)$, (2) $Cn(Cn(A)) = Cn(A)$, (3) and if $A, B \subset \mathcal{L}_1$, $A \subset B$, then $Cn(A) \subset Cn(B)$. Each of these axioms will be discussed in order to show how they model the most basic aspects of mental activity.

For axiom (1), suppose that A is a set of meaningful sentences upon which some form of mental active is performed. The result $Cn(A)$ represents all of those meaningful sentences that are obtained when that mental activity is performed on A. In order to perform the mental activity on A, the members of A must be mentally recognized. This is the first and absolutely necessary form of mental activity needed prior to application of Cn.

To codify this requirement within these axioms, it is required that the mental activity being performed allows $A \subset Cn(A)$. This process is necessary in almost all formal deduction and constitutes the process of stating an hypothesis as part of a deductive pattern. Indeed, if $A = Cn(A)$, then one would conclude that Cn yields no "new" deductive statements from A or, except for the recognition process, no "other meaningful" mental activity relative to $\mathcal{L}_1$ has occurred. It is also clear how to determine which strings of symbols are produced by this "other meaningful" mental activity that is distinct from the recognition requirement. These are those strings of symbols in $Cn(A)$ and not in A, i.e. members of $Cn(A) - A$.

Axiom (2) is called the idempotent axiom; and it is a symbolic representation for the concept that $Cn(A)$ should contain *all* members of $\mathcal{L}_1$ that are obtain from this specific mental activity, and simply applying Cn again will not alter the statement that *all* such members are obtained from the original application of Cn. Indeed, if $Cn(A)$ is a form of human deduction, then $Cn(A)$ is the *theory* obtained from the hypotheses A. The same form of human deduction Cn applied to a theory will not produce any new deductive results. Note that this aspect is dependent upon the Cn being considered. Different consequence operators (mental activities) applied to A can yield significantly different results, however.

Axiom (3) represents what the brain does sequentially prior to any additional mental activity. By way of illustration, suppose that $A, B \subset \mathcal{L}_1$, $A \subset B$, and A, B are sets of written strings of symbols. The mental activity represented by Cn is considered as fixed. From the axiom (2) notion, $Cn(A)$ represents all the symbol strings this mental active is able to express. Since $A \subset B$, then by this fixed mental activity applied to B, you can still restrict your activity, first, to A and obtain $Cn(A)$. Continued application to $Cn(A)$ will not yield any new results. But, next, you could get new results by applying the mental activity to the set $B - A$ and adjoin these results to $Cn(A)$. This is the exact approach when B is considered as a set of hypotheses to which scientific logic is applied. This type of sequential approach is codified by requiring $Cn(A) \subset Cn(B)$. Whether or not this might reduce the original $Cn(A)$ so that axiom (3) would not hold would depend upon another mental process, not considered as an aspect of this generalized consequence operator, that is performed after the $Cn(A)$ and $Cn(B)$ are obtained.

Although it may seem rather trivial, one of the more significant consequence operators is the *identity* operator, I. As the name indicates, I is defined on $\mathcal{L}$ [resp. $\mathcal{L}_1$] as follows: for each $A \subset \mathcal{L}$ [resp. $\mathcal{L}_1$], $I(A) = A$. This operator can be used to model cases where mental activity yields no "new" results and to construct consequence operators.

The Syntax Level

Relative to the origin and development of life, Gitt begins his information scheme not at this level of information but, rather, at the semantic level. Considering the equivalence class concept, this level corresponds to nonempty subsets of $\mathcal{L}_M$. The mental activity that produces this syntax level from $\mathcal{L}$ or even from the set of all language elements, does not appear to be a consequence operator. Under the most general possibility, suppose Sy is an operator that takes every subset of $\mathcal{L}$ and yields subsets of $\mathcal{L}$, which would be the first requirement for Sy to be a consequence operator. It seems reasonable to require that this operator should yield $Sy(\mathcal{L}) \subset \mathcal{L}_M$. However, unless $\mathcal{L} = \mathcal{L}_M$, which undoubtedly is not the case, then this reasonable requirement is contradicted by axiom 1. This does not mean that this possibility cannot be mathematically modeled, however.

There are operators that do model mathematically, in a general way, a mental process that would yield this required result and correspond to *Sy*. One of these is a mathematical finite or general selection operator (Herrmann, 1993, p. 100). The other is the deductive operator (Herrmann, 1993, p. 7). Although these do model, to some extent, certain very general mental activities, they are not consequence operators. Since the application of consequence operator theory to Gitt information theory is the purpose for the investigate presented here, the first application will assume that the sets $\mathcal{L}_1, \mathcal{L}_M$ have been previously obtained by means of mental-like activity modeled by one of these or possible some other similar mathematical operator.

The Semantic Level

The consequence operators discussed in Herrmann (1982, 1984, 1985, 1986a, 1986b, 1993, 1994a) are of a very special type and apply only to meaningful sets of readable sentences and yield only distinct meaningful results. One obvious aspect of this approach, however, was not required by these special consequence operators, but in a more general setting the idea of "content" (Herrmann, 1994c) was introduced. When restricted to human thought, a syntactically correct string of symbols can invoke a vast array of additional thoughts within the mind of an individual who perceives this symbol string. Due to differing education and training, these additional thoughts and images can differ from one individual to another. The term *content* has been used to express all of the effects a particular string of symbols has upon the mind of an individual who recognizes the string as a meaningful string of symbols. For this discussion, the content is restricted to those effects that can be represented by means of meaningful strings of symbols and is extended to include the general concept of mental activity. Any differences in content can be considered as the outcomes of different consequence operators. On the other hand, two distinct meaningful strings of symbols can produce the exact same content for a fixed consequence operator. Although the idea of content may be somewhat broad in character, the above restriction to effects that are expressible by meaningful strings of symbols corresponds to Gitt's third level of information, the *semantic level*. Hence, consider a fixed "semantics" generating consequence operator *Sm* defined on $\mathcal{L}_1$. Since *Sm* is a consequence operator, for a specific $y \subset \mathcal{L}_1$, the set of symbol strings $Sm(y) \subset \mathcal{L}_1$ is a complete semantic description obtainable from *Sm* and the language $\mathcal{L}_1$. This, of course, does not preclude a different semantic description using a different *Sm* or a different $\mathcal{L}_1$. From the above remarks, it is certainly possible that for two distinct $A, B \subset \mathcal{L}_1$, $Sm(A) = Sm(B)$. Hence, assume that *Sm* and the next two consequence operators are many-to-one operators; i. e. a specific value might be generated by more than one distinct member of $\mathcal{L}_1$.

Thus far, the term *information* has not been defined. Gitt (1997, p. 124) states that a general definition would be difficult to achieve. However, in accordance with the above, an analytical definition is possible for fixed *Sm* and $\mathcal{L}_1$. The consequence operator *Sm* can be viewed as a functional collection *Q* of ordered pairs. The semantic "information" is actually being displayed by this set *Q*. Thus *semantic information*, in this sense, is encoded by *Sm* and its corresponding *Q*. It is not a numerical measure but rather a set-theoretic concept. Indeed, using different semantic consequence operators Sm_1, Sm_2 one can investigate, as in Herrmann (1987), different strengths of semantic information. However, this additional investigation is not the purpose for this present article and will not be presented in any detail.

The Pragmatic Level

It is certainly possible that the semantic consequence operator is actually the identity operator. In this case, the actual mental activity *Sm* represents would be minimal. But, in many cases, the semantics includes strings of symbols which would require some sort of *action* to be undertaken; actions that, for a natural system, yield the objects and processes from which a natural system will be constructed. It is at this stage that more mental activity would ensue. The results of such mental activity Gitt calls the fourth level or *pragmatic level* of information.

The results of these actions can also be described in terms of meaningful strings of symbols as previous defined. Denote the corresponding consequence operator by *P*. Hence, the composition $P(Sm(A))$ yields the descriptive content of these actions. The actual new results of the actions are found in $P(Sm(A)) - Sm(A)$. Of course, *Sm* and *P* should be defined, at the least, on subsets of $\mathcal{L}_1$. Suppose that for a particular application of such operators, their axiom 1, 2, 3 behavior only holds for a special set $\mathcal{B}$ of subsets of $\mathcal{L}_1$ This is where the identity consequence operator might be used to extend, if necessary, the *Sm* and *P* to the entire set of all subsets of $\mathcal{L}_1$. For example, suppose that $\mathcal{L}_1$ is composed of the 26 alphabet symbols {a,b,..,z} each of which represents a meaningful string of symbols. Let $\mathcal{B} = \{\{a\}, \{a,b\}\} \cup \mathcal{D}$, where $\mathcal{D} = \{A \mid a \in A, A \subset \mathcal{L}_1\}$. Then $Q = \{(\{a\},\{a,b\})\} \cup \{(A, A \cup \{a,b\}) \mid A \in \mathcal{D}\}$ satisfies consequence operator axioms 1, 2, and 3. Now if $y \subset \mathcal{L}_1$ and $y \neq B$, for any $B \in \mathcal{B}$, then adjoin to Q the (y,y). It follows that extended Q represents a consequence operator on $\mathcal{L}_1$. Such consequence operators have a specific application to the concept of adjoining or removing a specific characterization from the description for a biological entity. Once again, the Gitt information associated with this level is encoded by Q.

The Apobetic Level

Once actions have actually been taken as they are encoded by $P(Sm(\mathrm{A}))$, that is the pieces of the "puzzle," so to speak, collectively exist, then to what plan or purpose does the $P(Sm(\mathrm{A}))$ correspond? Assuming that such a purpose is representable by characterizing and meaningful strings of symbols from $\mathcal{L}_1$, then to recognize that $P(Sm(\mathrm{A}))$ is not just composed of a set of disjoint descriptive statements but that these statements characterize a specific physical entity, plan, or purpose; yet another descriptive collection from $\mathcal{L}_1$ is obtained by means of another level of mental activity. For example, to recognize that $P(Sm(\mathrm{A}))$ produces a specific natural system that can be described by a sequence of symbol strings is such a mental activity. For a natural system, intuitively, the mental activity is a *combining together* of the necessary pieces obtained from the pragmatic level in order to produce a complete and identifiable natural system. Gitt calls this fifth level, a teleological information level, the *apobetic level* and it is modeled by a consequence operator T. Relative to a natural system, the result $T(P(Sm(\mathrm{A})))$ is always assumed to be a complete and consistent characterizing subset of $\mathcal{L}_1$. The term *complete* means that $T(P(Sm(\mathrm{A})))$ is maximal with respect to $\mathcal{L}_1$; i.e. $T(P(Sm(\mathrm{A})))$ contains only characterizing statements and if $x \in \mathcal{L}_1 - T(P(Sm(\mathrm{A})))$, then x is not a characteristic of the particular natural system being considered.

Following the application of the Sm, P, T consequence operators, a final mental-like operator, a *realism relation*, such as the one similar to that found in Herrmann (1993, p. 56) needs to be applied. For a given consequence operator Cn, this operator selects from the set of all subsets of $Cn(\mathrm{X})$ those collections of characteristics that can actually characterize an objectively real entity. However, due to axiom 1, this realism relation need only be applied after the application of the last consequence operator in a sequence of such operators, in this case after T, in order to identify the specific characterizing collections. The realism relation represents yet another necessary information level. It is tacitly assumed that this operator is applied.

Thus using the above notation and beginning at the syntactical level Gitt's entire sequence that represents the *internal transfer of information* with respect to a specific $\mathrm{A} \subset \mathcal{L}_1$ is captured by the sequence Sm, P, T of consequence operators with the final result described by $T(P(Sm(\mathrm{A})))$. One fact about such a sequence of consequence operators emerges from the theory of consequence operators. In Herrmann (1987, p. 4), the composition considered as a single operator of the Example 2.8 defined C' and Sm consequence operators is not itself a consequence operator. Thus, except under very special circumstances, the composition $(TPSm)$ cannot be considered as a single consequence operator. This implies a significant conclusion that the natural world mental activities modeled by Sm, P, T are usually *ordered* mental activities that are probably not representable by a single consequence operator. Except under very special circumstances, they need to be applied in the specific order indicated.

External Transmission of Information and Gitt's Theorems

There is considerable interest in the external transmission of Gitt information. This can be readily modeled by two sets of meaningful sentences A and A' and two sets of consequence operators defined on $\mathcal{L}_1$, the *sender* operators T, P, Sm and the *recipient* operators T', P', Sm'. A correspondence E between the sender and recipient is necessary and this would amount to another relation, not a consequence operator, such that $E(\mathrm{A}) = \mathrm{A}'$, $E(Sm(\mathrm{A})) = Sm'(\mathrm{A}')$, $E(P(Sm(\mathrm{A})) = P'(Sm'(\mathrm{A}'))$ and $E(T(P(Sm(\mathrm{A})))) = T'(P'(Sm'(\mathrm{A}')))$. Of course, it is possible that the sender and recipient are identical entities. In which case, E is but an identity. Further, the E correspondence indicates such things as whether the purpose intended by the sender is achieved by the recipient, for if this is the case, then $E(T(P(Sm(\mathrm{A})))) = T(P(Sm(\mathrm{A})))$. It is self-evident that many other aspects of the external transmission of Gitt information can be characterized by means of E and, further, all of the appropriate general results that have been established about consequence operators would hold for the ones used to model the Gitt concept of information. Recall that this mathematical model begins with the semantic level and not the statistical level that is the domain of information theory as originally envisioned by Shannon and Weaver (1964).

Some of the "Theorems" stated by Gitt (1997) are not inductively deduced by any strong logical argument nor are they obtained from a preponderance of empirical evidence. They are, more probably, axioms for his information theory concepts. However, many other Gitt theorems can be deduced directly from the above simple mathematical model. The following examples are deductive results that correspond to a some of Gitt's (1997) theorems. Theorem 7 (p. 65) states that the allocation of meanings to the set of available symbols is a mental process depending upon convention. This is but the consequence operators *Sm* and *Sm'*. Theorem 9 (p. 65) states that if the information is to be understood, the particular code must be know to both the sender and the recipient. This is but the fact that the $\mathcal{A}$ and $\mathcal{L}_1$ are the same for sender and recipient. Theorem 14 (p. 70) states that any entity, to be accepted as information, must entail semantics; it must be meaningful. Again this is *Sm* and *Sm'* applied to members of $\mathcal{L}_1$. Theorem 15 (p. 70) states that when its progress along the *chain of transmission events is traced backwards, every piece of information leads to a mental source, the mind of the sender*. Of course, this is but the inverse of E and its relation to the operators T, P, *Sm*. Theorem 16 (p. 71) states that if a chain of symbols comprises only a statistical sequence of characters, it does not represent information. This result is established since the consequence operators are defined first on the syntactical level since an appropriate logical operator that can be defined on the statistics level and that produces the syntax level is not a consequence operator. Further, Gitt's theorems 17, 19 - 22, 24 are established immediately and without difficulty. Gitt's other theorems can be established if one includes certain of his stated theorems as non-logical physical axioms, where the phrase *physical axiom* is used to differentiate inductive from deductive results.

Is Gitt information as presented within strings of symbols an actual and necessary requirement in order for the material universe to function? Gitt states (p. 29) that "The laws of nature are equally valid for living beings and for inanimate matter." His basic Theorem 1 (p. 47) is that "*The fundamental quantity information is a non-material (mental) entity*. It is not a property of matter, so that purely material processes are fundamentally precluded as sources of information." Then we have Theorem 23 (p. 79) "*There is no known natural law through which matter can give rise to information, neither is a physical process or material phenomenon known that can do this.*" There is the claim that all of the stated theorems are obtained by inductive logic using empirical evidence and as such are natural laws. This cannot be the case, however, with theorem 23 since the phrase natural law is part of the theorem statement itself and this theorem would be better classified as a statement within the philosophy of "information science." Further, Theorem 13 states that "Any piece of information has been transmitted by somebody and is meant for somebody. A *sender and recipient is always involved whenever and wherever information is concerned.*" Under Gitt's definition of natural law, Theorem 13 would need to be applicable to inanimate matter. But it is only applicable to "somebody." It appears likely that, at present, the only *direct* evidence that something like Gitt information is being used and transmitted is evidence relative to certain biological entities that exhibit mental activity and not evidence relative to purely inanimate material objects.

Most certainly one of the foremost advances in human intellectual development is the construction of various types of alphabets and written languages. Although actual neural processes are not being consider, some sort of material processes are taking place when an actual mental process is performed upon a set of symbol strings as an input and a set of such symbol strings is written down as an output. All of modern humankind's actual knowledge of natural law and the scientific descriptions for natural system behavior require a coded or symbolic information theory. The basic definition of the concept termed *natural laws* as these laws are applied today seems to require strings of symbols in order for various *relationships* that appear to exist between discipline dictionary named entities to be represented and comprehended by biological life-forms. There is always the possibility that there are relationships that exist between physical entities that cannot be expressed by any form of humanly comprehensible language.

Thus far, there is no evidence that the actual symbol strings that specify these relationships have been produced within the material world without the aid of a biological life-form. One might conclude that the behavior of a universe as a natural system would continue even if no intelligent life-forms exist. However, it is not the purpose of this article to discuss what might be a satisfactory definition of natural law. But, from the view point of how natural law is expressed today, information theory would only represent a *model* that mirrors certain aspects of natural system behavior that we seem unable to comprehend without its use. Indeed, if the genetic code and information theory is accepted as a reasonable explanation for how life could have come about from fundamental elements and fundamental natural law, then the genetic code and information theory still remains but a model for what could be natural processes that we can only comprehend through application of coding and information theory. This acceptance need not eliminate a search for other specific and fully materialistic processes that would achieve the same goal. On the other hand, nothing in this paper is intended to denigrate information theory or to imply that it is not significant to linguistics, to communication, to mental activity investigations, and as a model for natural system behavior. Those who specialize in information theory have and will continue to contribute significantly to scientific advances.

Nobel Laureate Louis deBroglie wrote:

> [T]he structure of the material universe has something in common with the laws that govern the workings of the human mind. (March and Freeman, 1963, p. 143)

This stated and rather obvious fact refers to human comprehension of natural system behavior. In order for us to comprehend and predict natural system behavior, human mental activity is used. General patterns associated with such mental activity would need to mirror the perceived general patterns associated with natural system behavior in order for us to predict by such mental activity specific natural system behavior. Being able to predict natural system behavior is the major application of natural law. Thus every reasonable prediction made and that is verified yields evidence that patterns of mental activity mirror patterns associated with natural system behavior whether the system be animate or inanimate. However, this still only implies that, in general, patterns associated with "information" are, at present, but models that can be an aid to comprehension and prediction.

An Application of Information as an Analog Model

First, one needs to define a mental process as modeled by consequence operators, say, as a *non-material process*. Relative to the actual physical world Yockey (1981, p. 26) reminds us that "Nothing which even vaguely resembles a code [alphabet] exists in the physico-chemical world." Assuming that the material universe of fundamental elements and natural laws has no actual symbolic code as a basic constituent, then, as with the case of ultralogics, it is, at the least, *rational to assume* that the processes and concepts associated with an information theory *analog model* is a required condition for our universe to exist and function. Although we may only be able to comprehend certain physical processes by means of information theory as an analog model this does not alter the statement that its application to the non-mental material world is an assumption that may never be fully established.

As stated previously, the *Sm*, *P*, *T* sequence of Gitt information consequence operators, from the natural world viewpoint, most probably are applied sequentially and taken as a composition they do not, in general, correspond to a single consequence operator. Each of these operators is a restriction to the natural world of three ultralogics ***Sm**, ***P**, ***T**, which are direct extensions of *Sm*, *P*, *T* to the nonstandard physical world as it is described in the Herrmann references. However, the sequential application of these three ultralogics must still hold if it holds for *Sm*, *P*, *T*. But, as will be shown, this is not the case if a special ultralogic and ultrawords (Herrmann, 1994a,b) are applied, where this ultralogic and ultrawords behave as an underlying control. In Gitt (1997, fig. 26, p. 137) relative to a scriptural interpretation, the A corresponds to a portion of the information contained in the genetic code in the DNA; *Sm*(A) is a description for the reading of the code, the meaning of the information in A, the rules, the instructions; *P*(*Sm*(A)) is a description for the actual protein synthesis in living cells, the construction of entire organisms, the realization of all biological functions; and the *T*(*P*(*Sm*(A))) is a description that would define a particular life-form by means of a complete description of its characteristics or as an integrated time ordered sequence of descriptions. (Note: The actual statements given in Gitt's figure 26 have been slightly extended.) From the viewpoint of models, these particular consequence operators and their corresponding ultralogics answer affirmatively the following question posed by one of the worlds foremost scientists. Hermann Weyl is credited with writing:

> Is it conceivable that immaterial factors having the nature of images, ideas, "building plans" also intervene in the evolution of the world as a whole?

Unfortunately, there is a fundamental error displayed in Gitt's figure 26 (p. 137). The figure indicates that the source (the Creator) of the information necessary for the generation of "life" is not within the scientific boundary. This, of course, is contradicted by a scriptural interpretation for the ultralogic and ultraword notion. Not only are ***Sm**, ***P**, ***T** ultralogics that yield the appropriate results, but using Theorems 7.3.1 and 7.3.4 for developmental paradigms, or as they are applied to general paradigms (Herrmann, 1993, p. 92) in the appropriate cases, for any $A \subset \mathcal{L}_1$ there exist four ultrawords w_{Sm}, w_P, w_T, w' and the single ultralogic $^*\mathbf{S}$ such that, when $^*\mathbf{S}$ is restricted to natural world symbol strings $\mathcal{L}_1$, we have (1) $^*\mathbf{S}(w_{Sm}) = Sm(A)$, (2) $^*\mathbf{S}(w_P) = P(Sm(A))$, (3) $^*\mathbf{S}(w_T)\} = T(P(Sm(A)))$, and $w_{Sm}, w_P, w_T \in {}^*\mathbf{S}(w')\}$. This illustrates the behavior of the ultralogic $^*\mathbf{S}$ as being fundamental in character, and how it exercises an additional control over the results of all other ultralogics that are extensions of specific natural world inner-logics such as *Sm*, *P*, *T*. In each case, these ultralogics behave, at the least, in the same manner as a consequence operator would behave; but, they can be classified in many ways as infinitely more powerful than natural world or standard consequence operators. Hence, there is a scientific description for the behavior of the Creator source and the natural world results generated by this source cannot be differentiated, by natural means, from the *Sm*, *P*, *T*. The scriptural statements that Gitt uses to identify the "sender" as the Creator are the exact type of statements mentioned in Herrmann (1984) where ultralogics and ultrawords are used as a model for God's creative and sustaining processes. This application should help to clarify the relation between information theory as presented by Gitt and previous work in the area of ultralogics and ultrawords.

Our final illustrations will indicate how Gitt information as encoded within consequence operators might apply to other aspects of genetic coding. Suppose that "complexity" is measured by the T description for the characteristics associated with a biological entity. Suppose that you are given $P(Sm(A))$ and two distinct consequence operators T_i, T_0. Then stating that $T_0(P(Sm(A)))$ is *as or more complex* than $T_i(P(Sm(A)))$ means that (I) $T_i(P(Sm(A))) \subset T_0(P(Sm(A)))$. Further. one could also write that $T_i(P(Sm(A)))$ is as or less complex than $T_0(P(Sm(A)))$. Notice that complexity in this case is a direct relative measure of T_i compared with another T_0. There are consequence operators that satisfy (I) for any $B \subset \mathcal{L}_1$. However, in general, can complexity be increased or decreased, at the T information level, by requiring the characterizing description to be obtained by choosing from two independent distinct characterizing descriptions $T_1(P(Sm(A)))$, $T_2(P(Sm(A)))$? Suppose T_1, T_2 are two distinct apobetic consequence operators applied to the same $P(Sm(A))$, but the results are distinctly different, one from another, at least with respect to one characterizing statement. The previous question is equivalent to the next question. Can it be assumed that (II) $T_0(X) = T_1(X) \cup T_2(X)$ defines a single apobetic consequence operator T_0 on nonempty $\mathcal{L}_1 \subset \mathcal{L}_M$ where $P(Sm(A)) \subset \mathcal{L}_1$ and the T_1, T_2 are independent? The term *independent* means that there is no relationship, accept for a set-theoretic identity relationship, between the T_1, T_2 that is expressible in terms of the set-theoretic operators used to obtain the theory of consequence operators. For example, suppose that such a consequence operator T_0 exists. Then a characteristic $X \in T_0(P(Sm(A)))$ is either a member of $T_1(P(Sm(A)))$ or a member of $T_2(P(Sm(A)))$ and a characteristic Y in $T_1(P(Sm(A)))$ or in $T_2(P(Sm(A)))$ is a characteristic in $T_0(P(Sm(A)))$ and no other characteristics exist in $T_0(P(Sm(A)))$. This implies that T_0 is a result that is as or more complex than the result obtained from T_1 or from T_2 or from both. Further, $T_0(P(Sm(A)))$ determines no other characteristics. Notice that if (II) holds, then (I) holds for each $i = 1, 2$.

In the appendix (with expanded proof) is a result that shows that (II) does not define a consequence operator if selection from two independent sets of characteristics is required. In particular, it is shown that if T_0 is a consequence operator defined on $\mathcal{L}_1$, then for each $X \subset \mathcal{L}_1$, $T_1(T_2(X)) = T_2(T_1(X))$; i.e. the composition of T_1, T_2 is a consequence operator and composition is a commutative process. This commutative requirement is a very special relationship that must exist between T_1 and T_2 and shows that no such consequence operator exists if independence is required. On the other hand, if (III) $T_1(T_2(X)) = T_2(T_1(X))$ for each $X \subset \mathcal{L}_1$, then the composition T_1T_2 is a consequence operator and (IV) $T_1(X) \cup T_2(X) \subset T_1(T_2(X))$ for each $X \subset \mathcal{L}_1$. Thus far, other than T_0 being a consequence operator, it has not been determined whether (III) is sufficient in order to replace in (IV) $\subset$ with $=$. If (III) is not sufficient for this purpose, then this is a rather interesting result since the required consequence operator composed of the composition of T_1, T_2 that is needed to yield the right hand side of (II), at the least, would also be capable of yielding $T_1(X) \cup T_2(X)$ possibility adjoined with other characteristics neither in $T_1(X)$ nor in $T_2(X)$. The following simple example shows that T_0 as defined need not be a consequence operator. Let $\mathcal{L}_1 = \{a,b,c\}$. In binary pair form, let $T_1 = \{(\emptyset,\{a\}),(\{b\},\{a,b,c\}),(\{a,b\},\{a,b,c\}),(\{b,c\},\{a,b,c\}),(\{a,b,c\},\{a,b,c\}), (\{a\},\{a\}), (\{c\},\{a,c\}),(\{a,c\},\{a,c\})\}$ and $T_2 = \{(\emptyset,\{b\}),(\{a\},\{a,b\}),(\{b\},\{b\}), (\{c\},\{b,c\}), (\{a,b\},\{a,b\}),(\{a,c\},\{a,b,c\}),(\{b,c\},\{b,c\}),(\{a,b,c\},\{a,b,c\})\}$. The relations T_1, T_2 represent consequence operators. But, $T_0(\emptyset) = \{a,b\}$ and $T_0(\{a,b\}) = \{a,b,c\}$. Hence, axiom (2) does not hold. Also notice that $T_1(T_2(\emptyset)) = T_1(\{b\}) = \{a,b,c\}$, but $T_2(T_1(\emptyset)) = T_2(\{a\}) = \{a,b\}$. On the other hand, for the above $\mathcal{L}_1$, if $T_1(\{a\}) = \{a,b\}$ and $T_2(\{a\}) = \{a,c\}$ and T_1, T_2 are extended as defined in the above section "The Pragmatic Level," then T_0 is a consequence operator.

There are other rather obvious ways to increase [resp. decrease] complexity. For example, relative to (I), there are consequence operators T_1, T_2 such that (V) for each $B \subset \mathcal{L}_1$, $T_2(B) \subset T_1(B)$. If $B = P(Sm(A))$, then this clearly yields the same or an increase [resp. decrease] in complexity. But, Theorem 2 in the appendix shows that if (V) holds, then we again have the same dependency statement that $T_1T_2 = T_2T_1$. On the other hand, a more direct way to give a possible increase in complexity is to let A, $B \subset \mathcal{L}_1$, $A \subset B$ and A and B are considered portions of the information in the genetic code in DNA. Then $T(P(Sm(A)))) \subset T(P(Sm(B)))$. For biological entities, this can be interpreted as predicting, what is rather obvious, that a possible greater complexity can be obtained from a fixed T by applying Sm to an increased portion of the information contained within the genetic code in the DNA, information that exists originally. However, in this case, to have a possible increase in complexity it is necessary that $P(Sm(A)) \neq P(Sm(B))$. Recall that, as in all previous cases, it is always assumed that $T(P(Sm(A)))$ and $T(P(Sm(B))$ are complete and consistent descriptions. Note that the technical results in this section also apply to the P and Sm operators. Scripturally, the special relationship between these consequence operators, with their encoded information, that is required to increase or decrease complexity gives further meaning to such statements as Hebrews 1:3.

In the above applications, it is assumed that consequence operators yield an ideal model for Gitt information theory. This model would need to be modified if modifications to the Gitt information or the information processes occurred. However, using Gitt's Theorem 1 (Gitt, 1997, p. 47), it is rational to assume that such modifications could only be produced through applications of additional mental activity. It is self-evident, that if consequence operators are used as a mathematical model for Gitt's concept of information and such an information scheme is accepted by the scientific community, then such results as presented here would most certainly require a rejection of the basic mechanisms that evolutionist claim yield a diversity of life-forms.

Conclusions

In this paper, there is developed a reasonable relation between consequence operators and information theory as presented by Gitt (1997). Although certain aspects of Gitt's theory may not be formulated in an acceptable manner, when re-formated, many of the theorems used for Gitt's theory are shown to be but deductive conclusions obtainable from the theory of consequence operators. It has also been shown that, relative to a scriptural interpretation of this Gitt information scheme, the source of life producing biological information corresponds to the scientifically describable concepts of ultralogics and ultrawords.

It is clear, however, that Gitt information theory, if it is accepted as a basic model for certain biological processes, may have a very profound effect upon those processes that evolutionists claim lead not only to "life" but also to the diversity of life-forms. If random *mutations* are random alterations in the characteristics of a biological entity that are, necessarily, not dependent upon the original biological characteristics, then this model would tend to disallow random mutations as a viable source of new evolutionary Gitt information. Mathematicians have produced an extensive theory of consequence operators and, hence, the mathematical model presented in this paper could be a very useful and additional tool in analyzing such claimed evolutionary processes.

Glossary

Analog model: In this paper, this is a theory that mimics a physical scenario—physical processes, physical characteristics, or natural system behavior—by application of objects or processes distinctly different from those discussed within the physical scenario.

Content: All of the effects a particular string of symbols has upon the mind of an individual who recognizes the string as meaningful.

Discipline dictionary: A collection of terms and phrases that has meaning for individuals who are associated with a specific discipline.

Equivalence class: Although this can be formally defined, for this paper, it is the collection of all meaningful strings of symbols that are adjudged to be "saying the same thing" or as having the "same relative meaning."

Independent consequence operators: This means that there is no relationship, accept for an identity relationship, between the consequence operators that is expressible in terms of the set-theoretic operators used to obtain the theory of consequence operators.

Inner-logics: The mental-like patterns that model behavior of or predict behavior for a specific natural system contained within our universe.

Internal transfer of information: This is information that is transferred between the Gitt information levels.

Model: A collection of statements that uses entities that need not correspond to the actual objects under consideration. The relationships that exist between specific entities within the statements are used to describe relationships that exist between the actual objects under consideration. It is these relations that mirror the behavior of the actual objects being discussed.

Natural system: A set or arrangement of physical entities within our universe that are so related as to form an identifiable whole.

Physical axiom: A statement using terms denoting assumed physical entities that is obtained by induction using empirical evidence.

Readable sentence: Another name for a word.

Representative: A single member of an equivalence class.

Ultralogics: An object that behaves like a mental process but is infinitely more powerful than any such process that models behavior that occurs within our universe.

Word: A nonempty finite string of symbols. Such a string can also correspond to visual and audio impressions.

Appendix

Theorem 1. *Let $\mathcal{C}$ be the set of all consequence operators defined on $\mathcal{L}_1$. Let $C_1,\ C_2 \in \mathcal{C}$ and, for each $X \subset \mathcal{L}_1$, let $C_0(X) = C_1(X) \cup C_2(X)$. If $C_0 \in \mathcal{C}$, then for each $X \subset \mathcal{L}_1$, $C_1(C_2(X)) = C_2(C_1(X))$.*

Proof. First notice that C_0 is a mapping on the power set of $\mathcal{L}_1$ into the power set of $\mathcal{L}_1$ and satisfies axioms 1 and 3 for consequence operators. Two auxiliary results need to be established using our three axioms.

Suppose that $X, Y \subset \mathcal{L}_1$ and $C \in \mathcal{C}$. From axiom 1, we have that $X \cup Y \subset X \cup C(Y) \subset C(X) \cup C(Y)$. Application of axiom 3, yields that (1) $C(X \cup Y) \subset C(X \cup C(Y)) \subset C(C(X) \cup C(Y))$. Since $X \subset X \cup Y$ and $Y \subset X \cup Y$, axiom 3 yields that $C(X) \subset C(X \cup Y)$, $C(Y) \subset C(X \cup Y)$. Consequently, $C(X) \cup C(Y) \subset C(X \cup Y)$. Applying axiom 2, it follows that (2) $C(C(X) \cup C(Y)) \subset C(C(X \cup Y)) = C(X \cup Y)$. It follows from (1) and (2) that (3) $C(X \cup Y) = C(X \cup C(Y)) = C(C(X) \cup C(Y))$.

For a given $C \in \mathcal{C}$, a $Y \subset \mathcal{L}_1$ is called a *C-system* if and only if $C(Y) \subset Y$, which is equivalent to $C(Y) = Y$. Note that for any consequence operator $C \in \mathcal{C}$, $\mathcal{L}_1$ is a C-system. Let $S(C)$ be the nonempty set of all C-systems for a given $C \in \mathcal{C}$. Suppose that (4) $\varnothing \neq \mathcal{A} \subset S(C)$ and (5) $X = \cap \{Y \mid Y \in \mathcal{A}\}$. Let arbitrary $Y \in S(C)$. Then $C(Y) \subset Y$. But fixed $X \subset Y$ implies that $C(X) \subset C(Y)$, which implies that $C(X) \subset \cap \{C(Y) \mid Y \in S(C)\} = \cap \{Y \mid Y \in S(C)\} = X$ since Y is arbitrary. Hence (6) $X \in S(C)$.

For each $C_1, C_2 \in \mathcal{C}$ define $C_1 \leq C_2$ if and only if for each $X \in \mathcal{L}_1$, $C_1(X) \subset C_2(X)$. The binary relation $\leq$ is a partial order on $\mathcal{L}_1$. The algebra $\langle \mathcal{C}, \leq \rangle$, along with other objects and relations is shown by Wójcicki (1970) to be a complete lattice. Our interest is in the structure of the least upper bound (the supremum) $C_1 \vee C_2$ for any $C_1, C_2 \in \mathcal{C}$. As shown by Wójcicki (1970, p. 276) for any $X \subset \mathcal{L}_1$, $(C_1 \vee C_2)(X) = Y_X = \cap \{Y \subset \mathcal{L}_1 \mid X \subset Y = C_1(Y) = C_2(Y)\}$. Thus the Y_X is by (6) a C_1-system and C_2-system. Indeed, intuitively the smallest (with respect to $\subset$) such common C-system. Now $X \subset Y_X$ implies that $X \subset C_1(X) \subset C_1(Y_X) = Y_X$ and $X \subset C_2(X) \subset C_2(Y_X) = Y_X$. Consequently, $C_1(X) \cup C_2(X) \subset Y_X$. Further, $C_1(X) \subset C_1(X) \cup C_2(X)$, $C_2(X) \subset C_1(X) \cup C_2(X)$. Define for each $X \subset \mathcal{L}_1$, $C_0(X) = C_1(X) \cup C_2(X)$ and assume that $C_0 \in \mathcal{C}$. First notice that $C_1 \leq C_0$, $C_2 \leq C_0$ and $C_0 \leq C_1 \vee C_2$. Thus $C_0 = C_1 \vee C_2$. This implies that for arbitrary $X \subset \mathcal{L}_1$, $C_1(X) \cup C_2(X) = Y_X = C_1(Y_X) = C_2(Y_X)$ by (6). Therefore, $C_1(C_1(X) \cup C_2(X)) = C_1(Y_X) = C_2(Y_X) = C_2(C_1(X) \cup C_2(X))$. From (3), $C_1(X \cup C_2(X)) = C_2(X \cup C_1(X))$. But, axiom 1 yields that $C_1(C_2(X)) = C_2(C_1(X))$ and the result follows.

Theorem 2. *Let $\mathcal{C}$ be the set of all consequence operators defined on $\mathcal{L}_1$. Let $C_1, C_2 \in \mathcal{C}$. Then $C_2 \leq C_1$ if and only if $C_1C_2 = C_2C_1 = C_1$.*

Proof. As shown in Herrmann (1987, p. 7), $C_2 \leq C_1$ if and only if the composition $C_1C_2 = C_1$. Assume that $C_2 \leq C_1$. Then for arbitrary $X \subset \mathcal{L}_1$, $C_1(C_2(X)) = C_1(X) \subset C_2(C_1(X)) \subset C_1(C_1(X)) = C_1(X)$ implies $C_1(C_2(X)) = C_2(C_1(X)) = C_1(X)$. The converse is obvious and this completes the proof.

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