A Review of the Lynden-Bell/Choloniewski Method for Obtaining Galaxy Luminosity Functions

Part I

Jake Hebert and Jason Lisle*

Abstract

In the biblical worldview, Earth is not just another planet, but the unique world where God placed those who are made in His image. For this reason, some creationists have speculated that our world might have a privileged location in the physical universe. For example, some have suggested that the solar system may be near the center of a sequence of concentric shells of high galaxy number density, accounting for the "quantized redshifts" seen in cosmic surveys. In order to rigorously test for this possibility, it is necessary to first determine the true spatial distribution of galaxies in our local neighborhood. In order to do this, however, it is necessary to correct for the fact that some dim galaxies are too faint to be seen, an effect that becomes more severe with increasing distance. This correction is often obtained via a *luminosity function*, which gives the fraction of galaxies that fall within a selected intrinsic brightness range. Recently, the Institute for Creation Research has done additional work in this area. In order that the creation science community may be able to intelligently critique future galactocentric claims, this review presents a detailed description of one method for obtaining the luminosity function.

Introduction

The *cosmological principle* of secular cosmology is the assumption that on the largest distance and angular scales, there are no special places or directions in the cosmos (Gabrielli et al., 2010, p. 31; Humphreys, 1994, pp. 14–21). Moreover, as noted by Hartnett (2007, p. 75), secular scientists have in the past been adamant that neither Earth nor the Milky Way galaxy could be "special" in the cosmic scheme of things. In the biblical worldview, there is good reason to question this assumption, given the importance of Earth in God's plan of redemption. Hence, creation scientists do not uncritically accept the *cosmological principle* of big-bang cosmology. The

* Jake Hebert and Jason Lisle, Institute for Creation Research, Dallas, TX, contact@icr.org

Accepted for publication February 11, 2016

biblical worldview may well allow for this principle, but it does not require it.

There has long been interest within the creation community regarding the possibility that our Milky Way galaxy might be located within a special or preferred location in space (Humphreys, 2002; Hartnett, 2005). For instance, the evidence of concentric rings of high galaxy density roughly centered on the location of our Milky Way galaxy is very unexpected within big-bang cosmology. However, there is some question as to whether this effect is real. Maps of galaxy positions are generated from Earth-based observations, which may lead to observer-centered distortions giving the false appearance of Earth-centered structures, such as the "fingers of God" effect discovered in the late 1950s. Observer-centered biases must therefore be carefully quantified in order to assess the legitimacy of any galactocentric claims. One such effect is the *Malmquist bias*: the systematic drop in our ability to detect faint galaxies at progressively greater distances. The purpose of this paper is to clearly and concisely explain one of the techniques that may be used to determine the most likely true distribution of galaxies, given the galaxies that are actually observed. The Institute for Creation Research has recently done significant additional work in this area and one of us (Lisle) has devised a novel approach to determining galaxy luminosity functions (Lisle, 2016). Because these methods are often not clearly explained in the technical literature, it is our hope that this review will help familiarize the creation science community with these methods so that future results may be intelligently critiqued.

The Malmquist Bias

In a flux-limited (or magnitude-limited) survey, the number of galaxies included in the survey is determined by the minimum amount of light per unit area (flux) that can be detected by the survey instrument. In such a flux-limited survey, the observed number density of galaxies decreases with increasing distance from Earth (Strauss and Willick, 1995, p. 54). This is due to the fact that the apparent brightness of a galaxy diminishes with increasing distance. Thus, at close distances, both faint and bright galaxies are detectable. But at greater distances, only bright galaxies can be detected. Hence, bright objects tend to be overrepresented in the survey simply because they are easier to detect, and this effect becomes more pronounced with increasing distance. This Malmquist bias is named after the Swedish astronomer Gunnar Malmquist (Heydari-Malayeri, 2014). The Malmquist bias must be taken into account when attempting to determine the true distribution of galaxies.

To compensate for the Malmquist bias, one needs to obtain a selection function *S(r)* that gives the fraction of galaxies that will be observed at a particular distance relative to the number that actually exist at that distance. Conceptually, this fraction is given by the observed number density of galaxies at a given distance divided by the true number density of galaxies:

$$S(r) = \frac{\rho_{obs}(r)}{\rho_{true}(r)} \tag{1}$$

The observed number density is known, since this is simply the number of galaxies detected in a volume of space in a given survey. If the selection function is also known, then it is possible to obtain the true galaxy number density as a function of distance, thereby removing the Malmquist bias.

In order to obtain this selection function, it is necessary to know something about the distribution of galaxy brightnesses so that we can estimate how many of the fainter galaxies we are missing at larger distances. Specifically, we must find what fraction of galaxies fall within a selected range of intrinsic luminosity. This histogram of galaxy brightnesses is called the *luminosity function*. We begin our discussion of galaxy luminosities by addressing the relevant terminology.

Luminosity and Magnitudes

The apparent magnitude m of an astronomical object is a measure of its apparent brightness as it appears in our sky. Apparent magnitudes use a logarithmic scale that allows astronomers to describe an enormous apparent brightness range with relatively small differences in magnitude values. For instance, the apparent magnitude of the Sun is -26.7, while the dimmest object that can be detected by the human eye has an apparent magnitude of about +6.0 (Freedman and Kaufmann, 2002, p. 427). As can be seen from these two examples, brighter objects have smaller apparent magnitudes, while dimmer objects have higher apparent magnitudes. The apparent magnitude scale has been defined in such a way that a magnitude difference of 5 corresponds to a brightness ratio of exactly 100 (i.e., an object with $m = 1.0$ is 100 times brighter than an object with $m = 6.0$). It should also be noted that m is frequency-dependent, since galaxies emit different intensities of light at different frequencies. In galaxy surveys, apparent magnitudes are generally measured for a given bandwidth.

The *absolute* magnitude M is a measure of an object's intrinsic brightness. Specifically, it is defined as the apparent magnitude that an object would have at a distance of 10 parsecs (1 parsec = 3.26 light-years). As is the case with apparent magnitudes, objects of greater intrinsic brightness have smaller values of M than do intrinsically dimmer objects.

The redshift z is the fractional change in the wavelength of a galaxy's light between emission and detection, which is generally attributed to the expansion of the universe.

The quantity $d_L(z)$ is the (redshift-dependent) luminosity distance to the galaxy, where the luminosity distance is a distance measurement for which light obeys the inverse-square law.

The relationship between the absolute and apparent magnitudes of an object and the object's luminosity distance (measured in parsecs) is given by Freedman and Kaufmann (2002, p. 429) and Hogg et al. (2002):

$$m = M + 5\log_{10}\left(\frac{d_L(z)}{10\text{ pc}}\right) = M + 5\log_{10} d_L(z) - 5 \quad (2)$$

We define the distance modulus μ such that

$$\mu = 5\log_{10}\left(\frac{d_L(z)}{10\text{ pc}}\right) = m - M \quad (3)$$

Eqs. (2) and (3) as written fail to take into account the need for a so-called *K-correction*. This complication is addressed later in the paper.

If the luminosity distance is expressed in mega-parsecs (Mpc), then Eq. (2) becomes

$$m = M + 5\log_{10} d_L(z) + 25 \quad (4)$$

The Luminosity and Selection Functions

The luminosity function is defined such that $\phi(M)dM$ is the estimate for the *true* (not just observed) number of galaxies per unit volume having absolute magnitudes between M and $M + dM$. At this point, it is necessary to briefly discuss the concept of comoving distances and volumes.

In cosmological studies, a *comoving distance* is one in which the expansion of the universe has been "factored out" (Bergström and Goobar, 2008, pp. 62, 200). Hence, if one were to imagine a hypothetical universe in which galaxies had no velocities other than those due to the expansion of the universe (i.e., no *peculiar velocities*), then the comoving distances between galaxies would remain constant. Since comoving distances (and volumes) remain unaffected by the expansion, comoving distances are optimal distance measurements for studying the large-scale distribution of galaxies. Hence, when determining the luminosity function, comoving volumes should be used when computing number density. Hogg et al. (2002) present a number of standard formulae for calculating comoving distances. However, it should be noted that these formulae have been derived under the assumptions of a homogeneous and isotropic universe (standard assumptions also used by big bang advocates). In a spatially "flat" universe, in which parallel lines never meet or diverge, the comoving line-of-sight distance d_C to a galaxy is related to the luminosity distance and redshift by

$$d_C(z) = \frac{d_L(z)}{(1+z)} \quad (5)$$

When attempting to discern possible patterns in the spatial distribution of galaxies, one would prefer to keep such an analysis as "assumption-free" as possible. Hence, it is understandable that one might object to the use of standard assumptions for the distance formulae in such an analysis. Unfortunately, it is simply not possible to convert redshifts to distances without making *some* assumptions. However, the use of standard parameters for the distance formulae in such an analysis is not as problematic for our purposes as one might initially suspect. For instance, suppose that one could somehow discern the true comoving distances to all the galaxies within a survey without making any cosmological assumptions whatsoever. Then suppose we were to calculate these comoving distances using Eq. (5). Even if these distances were to disagree somewhat, there would still be a one-to-one correspondence between the calculated and true comoving distances. Hence, any "peaks" in galaxy density that appear when the density is plotted as a function of redshift will *also* appear when the density is plotted as a function of comoving distance. So even if these calculated distances are distorted somewhat due to incorrect cosmological assumptions, concentric rings of high galaxy density, if they exist, should still be apparent when the galaxy density is plotted as a function of distance.

Once the luminosity function has been found, the selection function may be obtained. Since the redshift z is a proxy for distance, the selection function is often expressed in terms of redshift:

$$S(z) = \frac{\int_{M_{\min}}^{\min\left[M_{\max},\, M_{\lim(z)}\right]} \phi(M)\,dM}{\int_{M_{\min}}^{M_{\max}} \phi(M)\,dM} \quad (6)$$

The denominator in Eq. (6) is, for a given redshift z, the true number of galaxies per unit volume between the magnitude limits M_{min} and M_{max} of the survey, while the numerator is the observed number of galaxies per unit volume within those same magnitude limits and at that particular value of z. Hence Eq. (6) can be thought of as the fraction of galaxies we are detecting, at a particular redshift z, compared to the actual number that exist.

For the best estimate of the selection function, one should choose the upper and lower limits on the absolute magnitudes (M_{min} and M_{max}) to include as many galaxies as possible. For purposes of this explanatory paper, we will ignore certain complications that arise in real galaxy surveys, although these complications are addressed in follow-up papers (Hebert and Lisle, 2016; Lisle, 2016). For instance, real galaxy surveys sometimes contain a small number of outlying galaxies having anomalously high or low absolute magnitudes, magnitudes that are either extremely rare or perhaps even simply erroneous. Likewise, traditional methods of finding the luminosity function, including the Lynden-Bell/Choloniewski (LBC) method, assume a "hard" apparent magnitude limit cutoff; i.e., apparent magnitudes less than or equal to a limiting apparent magnitude m_{max} can be observed, while apparent magnitudes greater than m_{max} cannot. In real galaxy surveys, however, the apparent magnitude cutoff is "fuzzy"—the vast majority of galaxies within the survey have apparent magnitudes less than m_{max}, but a small fraction of galaxies have apparent magnitudes greater than this number. Since traditional methods presuppose a "hard" apparent magnitude cutoff, the best choice for m_{max} when using these methods is actually a number that is *less* than the very dimmest apparent magnitude in the survey. For purposes of this discussion, however, we ignore these complications. Instead, we assume that our hypothetical galaxy survey contains no anomalously high or low absolute magnitudes, and we assume the existence of a "hard" apparent magnitude cutoff of m_{max}. Once the basic theory behind the LBC method has been described, necessary modifications to the method demanded by conditions in real galaxy surveys are discussed in follow-up papers (Hebert and Lisle, 2016; Lisle, 2016).

The number $M_{lim(z)}$ in our expression for $S(z)$ is the absolute magnitude of the faintest galaxy that could in principle be detected at the redshift z. This is computed for each value of z by taking the upper limiting apparent magnitude of the survey (denoted as m_{max}) minus the distance modulus $\mu(z)$ corresponding to that redshift, as given in Eq. (3). This is expressed as follows:

$$M_{\lim(z)} = m_{\max} - 5\log_{10}(d_L(z)) - 25 = m_{\max} - \mu(z) \tag{7}$$

The limiting apparent magnitude m_{max} is determined by the limitations of the detector since it can obtain a reliable spectrum (necessary for redshift estimation) only over a particular range of apparent magnitudes. For galaxies to be included in the SDSS survey, their *r*-band Petrosian apparent magnitude must be lower than 17.77 (Strauss et al., 2002). Hence one might expect m_{max} to be equal to 17.77 for the SDSS survey. However, the final estimate for this limiting apparent magnitude is not exactly 17.77 due to some complicating factors. These factors, as well as more detailed discussions of different magnitude systems, are discussed in later papers (Hebert and Lisle, 2016; Lisle, 2016).

Assumptions of the Method

When doing any kind of scientific analysis, it is good to be aware of one's underlying assumptions. The LBC method involves a number of implicit assumptions. For instance, the denominator of Eq. (6) is said to give the true number of galaxies per unit comoving volume for any redshift z, even though the denominator is formally independent of z. This implies that the luminosity function is independent of redshift (or equivalently, of distance). Hence one does not expect galaxies of a particular brightness to be preferentially located at any particular distance or distances. Although dimmer galaxies are much more numerous than brighter galaxies (Sparke and Gallagher, 2010, p. 396), one does not expect dim galaxies to be any more or less likely to be located at a particular distance than galaxies of any other brightness. This assumption, though not strictly correct, is widely accepted as a good first approximation (Gabrielli et al., 2010, pp. 221, 291). It should also be noted that there are ways to test the validity of this assumption (Gabrielli et al., 2010, pp. 293–297).

Because the Lynden-Bell/Choloniewski method implicitly assumes a "hard" cutoff for the survey's limiting apparent magnitude, this implies that at a particular redshift z, any galaxy having an absolute magnitude less than $M_{lim(z)}$ will automatically be detected. To put it another way, the method assumes that there is a 100% probability that any galaxy at a redshift z having magnitude $M \leq M_{lim(z)}$ will be detected, but there is a 0% probability that any galaxy with $M > M_{lim(z)}$ will be detected. While this may seem perfectly reasonable, it is not necessarily true in real galaxy surveys, for reasons discussed in our follow-up papers (Hebert and Lisle, 2016; Lisle, 2016). Nonetheless, the approximation is workable. We have also assumed that no true galaxies have been misidentified as something other than a galaxy (100% survey *completeness*) and that no nongalaxy objects have been misidentified as galaxies (100% survey *efficiency*). This too is a good approximation, but it is not exact. A brief discussion of survey completeness and efficiency is proved in Ball and Brunner (2010).

One of the better-known methods for obtaining the luminosity function is the Lynden-Bell/Choloniewski (LBC) method, originally proposed by Lynden-Bell (1971) and later modified by Choloniewski (1987). Willmer (1997) has argued, on the basis of Monte Carlo simulations, that of the conventional methods for finding galaxy luminosity functions, the LBC method provides the most robust estimate of the luminosity function's shape, although another method, the stepwise maximum-likelihood method, devised by Efstathiou, Ellis, and Peterson (1988) seems to be more popular.

The LBC Method: Introduction

For purposes of this discussion, we are considering a galaxy survey consisting of N_{obs} observed galaxies. The absolute magnitude M, the apparent magnitude m, the redshift z, and the distance modulus μ are all assumed to be known for each of the N_{obs} galaxies. The survey's smallest value of μ is denoted by μ_{min} and the largest value of μ is denoted by μ_{max}.

From its original derivation, the LBC assumes that no two galaxies in the survey have exactly the same absolute magnitude. This is not true of the SDSS survey; due to its limited precision, a substantial fraction of galaxies share absolute magnitudes. For the time being, this complication is ignored as we present Lynden-Bell's and Choloniewski's original derivations. However, this complication does not affect their result, as shown in a second paper in this series (Hebert and Lisle, 2016). The galaxies are sorted by absolute magnitude in order of decreasing brightness, such that $M_{k+1} > M_k$. As noted earlier, we assume a "hard" cutoff for the survey's limiting apparent magnitude, which is denoted by m_{max}.

Consider a plot of the μ versus M values for each of the N_{obs} galaxies in the survey. Figure 1 illustrates this for a small hypothetical sample of ten observed galaxies. Of course, a real galaxy survey can contain many thousands of galaxies. The brightest galaxies (those with the smallest absolute magnitude values) are located toward the left-hand side of the figure. We have chosen the survey to have lower and upper distance modulus limits of μ_{min} and μ_{max}, respectively. Once these limits have been chosen, a particular galaxy within the survey will have the smallest absolute magnitude M_{min}, and another galaxy will have the largest absolute magnitude M_{max}. Note that galaxies above the diagonal m_{max} line cannot be detected because they are too faint to be seen. Also note that given the hard apparent magnitude cutoff m_{max} and the distance modulus limits of the survey, the largest absolute magnitude that can theoretically be contained within the survey is $M = m_{max} - \mu_{min}$. As noted earlier, however, a real galaxy survey can sometimes contain galaxies with absolute magnitudes greater than this theoretical upper limit. The reason for this counterintuitive result is presented in Hebert and Lisle (2016). For the time being, we consider only those galaxies that are brighter than this theoretical upper limit.

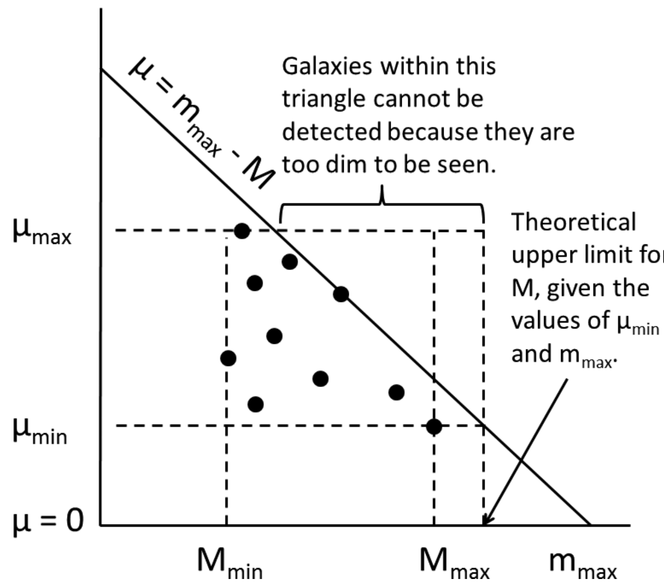

Figure 1. Plot of distance modulus μ versus absolute magnitude M for a simulated magnitude-limited survey containing ten observable galaxies. Of course, a real galaxy survey may contain many thousands of such galaxies. Note that the diagonal line is defined by the limiting apparent magnitude m_{max} of the survey, such that no galaxies are observed within the triangle above the diagonal line. As discussed in the text, it is assumed that these absolute magnitudes have been *K*-corrected, and that the distance moduli have already been transformed according to Eqs. (18) and (19).

Lynden-Bell's Original Method: The Basic Concept

The basic premise behind the method is that, as is usually assumed, the luminosity function $\phi(M)$ is independent of z (or equivalently, independent of μ).

Consider an arbitrary value of absolute magnitude M_k located between the bounds of the survey M_{min} and M_{max} (Figure 2). We can count the observed number of galaxies within the thin shaded rectangle centered at M_k. This thin shaded rectangle is itself contained within a large shaded rectangle, and we can count the number of observed galaxies within the large shaded rectangle, as well. We denote the number of galaxies within the small rectangle as $dX(M_k)$, and we denote the number of galaxies within the large shaded rectangle as $C(M_k)$. Note that all the galaxies we have counted are below the value of μ at the point where the diagonal line defined by $\mu = m_{max} - M$ intersects M_k, so that we are only considering galaxies between the horizontal lines defined by $\mu = \mu_{min}$ and $\mu =$

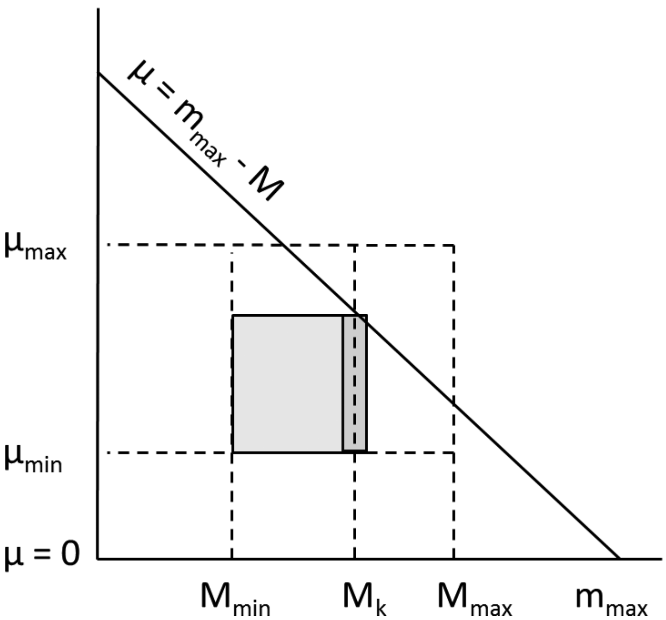

Figure 2. Because the tops of both the thin and large gray rectangles lie below the diagonal line, the counted numbers of observed galaxies in both rectangles should be equal to the true numbers of such galaxies. The number of observed galaxies within the thin rectangle compared to the number of observed galaxies within the large rectangle is denoted in this paper as *Fraction$_1$*.

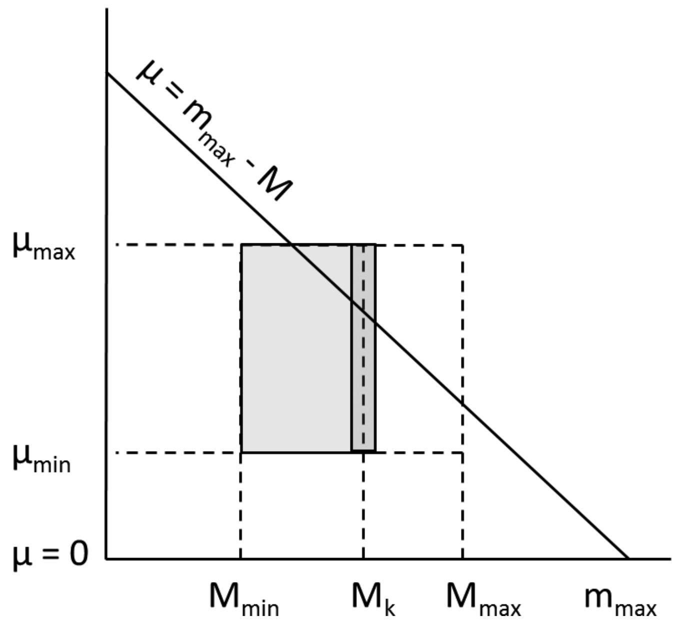

Figure 3. Although the true distribution of galaxies *$\phi(M)$* is unknown, if one assumes that *$\phi(M)$* is independent of distance (or *μ*), then the ratio of the true number of galaxies within the thin rectangle compared to the true number of galaxies within the large rectangle (denoted as Fraction2) should be equal to *Fraction$_1$*, as described in Figure 2.

$m_{max} - M_k$. Because all the galaxies we are counting are below the diagonal line (and hence should be visible), we should, in theory, not be overlooking any galaxies and should be counting 100% of the galaxies that are actually present within the two rectangles. In other words, the observed number of galaxies within the two rectangles should be equal to the true number of galaxies. Note also that both the thin and large rectangles have the same height, which means they are both defined by the same maximum and minimum distance moduli values. Or equivalently, they are characterized by the *same* comoving volumes in physical space. We can define the fraction (or percentage) of galaxies within the thin rectangle compared to the large rectangle as

$$\text{Fraction}_1(M_k) = \frac{\text{Galaxies/volume (thin rectangle)}}{\text{Galaxies/volume (large rectangle)}} = \frac{\text{Galaxies in thin rectangle}}{\text{Galaxies in large rectangle}} = \frac{dX(Mk)}{C(Mk)} \tag{8}$$

Now consider two similar rectangles, but which extend all the way from $\mu = \mu_{min}$ to $\mu = \mu_{max}$ (Figure 3). Remember that $\phi(M)dM$ is *defined* to be the true number of galaxies per unit volume between M and $M + dM$. Hence, we can define in a similar fashion a second fraction as

$$\text{Fraction}_2(M_k) = \frac{\text{Galaxies in thin rectangle}}{\text{Galaxies in large rectangle}} = \frac{\phi(M_k)dM}{\int_{M_{\min}}^{M_k} \phi(M)dM} \tag{9}$$

Remember also that $\phi(M)$ is assumed to be independent of z (or equivalently, of μ). This means that galaxies within the bin centered on M_k should make the same fractional (or percentage) contribution to the cumulative number of galaxies per unit volume (such that $M \leq M_k$) *regardless of the size of the comoving volume being examined*. In other words, we are assuming that however many galaxies in our survey have $M \approx M_k$, these galaxies are more or less uniformly distributed throughout the comoving volume of the survey. So if galaxies with $M \approx M_k$ contribute 0.5% to the cumulative number of galaxies per unit comoving volume (such that $M \le M_k$) for values of μ ranging between $\mu = \mu_{min}$ and $\mu = m_{max} - M_k$, then they should *also* contribute ~0.5% to the cumulative number of galaxies for μ between μ_{min} and μ_{max}. Hence, if this assumption is true, then $\text{Fraction}_1(M_k) = \text{Fraction}_2(M_k)$:

$$\frac{dX(M_k)}{C(M_k)} = \frac{\phi(M_k)dM}{\int_{M_{\min}}^{M_k} \phi(M)dM} \tag{10}$$

This is the fundamental assumption behind the method. Let us now define $\psi(M_k)$ to be the cumulative luminosity function for $M_{min} \le M \le M_k$:

$$\psi(M_k) = \int_{M_{\min}}^{M_k} \phi(M)dM \tag{11}$$

such that

$$d\psi = \phi(M)dM \tag{12}$$

is the differential contribution to this luminosity function. We may thus rewrite Eq. (10) as

$$\frac{dX(M_k)}{C(M_k)} = \frac{d\psi(M_k)}{\psi(M_k)} \tag{13}$$

Note that because M_k is an arbitrary value of M, it may be treated as a "dummy" variable should one integrate Eq. (13) up to some specific value of M_k, say $M_{k'}$. Performing this integration from $M = M_{min}$ to $M_k = M_{k'}$ yields

$$\psi(M_{k'}) = A\exp\left(\int_{M_{\min}}^{M_{k'}} \frac{dX}{C} \right) \tag{14}$$

After careful consideration of the manner in which a single galaxy would contribute to this integral, Lynden-Bell (1971) converted this expression into a discrete form. Replacing $M_{k'}$ with M' to match his notation we have:

$$\psi(M') = \int_{M_{\min}}^{M'} \phi(M)dM$$

$$= A \prod_{i:}^{M_{\min}<M_i<M'} \left(\frac{C^-(M_i)}{C^-(M_i)+1} \right) \tag{15}$$

Eq. (15) gives an expression for the (unnormalized) cumulative luminosity function in terms of $C^-(M_i)$, which is defined to be the number of points inside the large rectangle of Figure 2 but excluding the contribution of the i^{th} galaxy itself. Note that for $i = 1$, the way in which we have defined $C^-(M_i)$ implies that $C^-(M_1 = M_{min}) = 0$. To prevent ψ from being equal to 0, Lynden-Bell imposed the requirement that the quantity in parentheses in Eq. (15) be equal to 1 for $M_i = M_{min} = M_1$. Because Choloniewski's version of the method is in some ways more straightforward, we will not discuss this version of the method any further.

A Complication: The *K*-Correction

However, before discussing Choloniewski's modification to the Lynden-Bell method, it is necessary to address a complication when using Eqs. (2), (3), and (4) that was not explicitly addressed by either Lynden-Bell or Choloniewski. The apparent magnitude m in Eq. (2) has been determined through a bandwidth in the observer's reference frame. However, determination of the galaxy's true intrinsic brightness should be performed in the galaxy's rest frame. Since the light from the galaxy is being detected through a filter of limited bandwidth, red- or blue-shifting of this light will cause the portion of the galaxy's spectrum that can be detected by the filter to be shifted partly into or out of the filter's bandwidth, leading to a distortion in the calculated absolute magnitude M. In order to correct for this distortion, the z-dependent K-correction has been defined (Hogg et al., 2002) such that

$$m_{\text{observed frame}} = M_{\text{emitted frame}} + \mu + K(z) \tag{16}$$

A direct calculation of the K-correction requires integration of a galaxy's flux spectral density (energy per unit time per unit area per unit frequency). Such calculations are computationally unrealistic for galaxy surveys containing hundreds of thousands of galaxies. However, much faster analytical techniques can estimate the appropriate K-corrections. For instance, Chilingarian, Melchior, and Zolotukhin (2010) have published a technique (as well as an online K-calculator and codes at http://kcor.sai.msu.ru/

getthecode/) for such a purpose. Their technique requires the galaxy redshift z and the galaxy color, which is defined as the difference in apparent magnitudes measured in two different bandwidths. If one measures the luminosity distance in megaparsecs (Mpc), the expression for the K-corrected absolute magnitude becomes:

$$M_{\text{corrected}} = M_{\text{emitted frame}} = m_{\text{observing frame}} - 5\log_{10} d_L(z) - 25 - K(z, \text{color}) \quad (17)$$

Furthermore, Eq. (16) is now the defining relationships between M, m, and μ, rather than Eq. (3). However, Choloniewski did not include a K-correction in his discussion of the method. This omission may be ameliorated by using Eq. (16) to obtain the K-corrected absolute magnitude for each galaxy and by making a coordinate transformation

$$\mu' = \mu + K_{\text{avg}}(z) = m_{\text{observed frame}} - M_{\text{emitted frame}} = m_{\text{observed frame}} - M_{\text{corrected}} \quad (18)$$

where $K_{avg}(z)$ is the average K-correction of galaxies in our survey at redshift z. Details on obtaining K_{avg} are provided in Lisle (2016). Using the μ' coordinate frame, the K-correction appears to vanish as it is absorbed into the distance modulus. Hence the same relationship exists between μ', $M_{corrected}$, and m in Eq. (18) as between μ, M, and m in Eq. (16). Thus we may continue to use Choloniewski's method, despite this complication, provided that we use K-corrected absolute magnitudes and replace μ with μ'. Note that this coordinate frame transformation is an approximation that accommodates the average K-correction of all galaxies at redshift z, though individual galaxies may have somewhat different K-corrections due to their various colors.

We will follow Choloniewski's notation in this discussion of his method and will therefore omit the prime symbol from the distance modulus. But bear in mind that from this point on we are assuming that each absolute magnitude M has already been K-corrected and that each value of μ has been replaced by $\mu' = \mu + K_{avg(z)}$. Once these new values of μ have been determined, they may be plotted on a chart (as in Figure 1). Note that μ_{min} and μ_{max} must also be converted to the μ' coordinate system as follows:

$$\begin{aligned} \mu'_{\min} &= \mu_{\min} + K_{\text{avg}}(z_{\min}) \\ \mu'_{\max} &= \mu_{\max} + K_{\text{avg}}(z_{\max}) \end{aligned} \quad (19)$$

Choloniewski's Modification to the Method

Choloniewski (1987) modified Lynden-Bell's original method in order to obtain a normalized (noncumulative) differential version of the luminosity function. In considering Figure 1, note that the apparent number density of observed galaxies in the μ-M plane (actually, the $\mu' - M_{corrected}$) plane, which we shall call N_{app}, may be expressed as a sum of two-dimensional Dirac delta functions

$$N_{app}(M, \mu) = \sum_{k=1}^{N_{obs}} \delta(M - M_k, \mu - \mu_k) \quad (20)$$

such that the apparent galaxy number density at a given location in the M-μ plane is zero if a galaxy is not present, but infinite if one is. The apparent density may also be expressed as

$$N_{app}(M, \mu) = N_{\text{true}}(M, \mu)\Theta(m_{\max} - m) \quad (21)$$

where $N_{true}(M, \mu)$ is the true density of galaxies in the M-μ plane, multiplied by a Heaviside "step" function, defined as

$$\Theta\left[m_{\max} - m\right] = \Theta\left[m_{\max} - \left(\mu + M\right)\right] = \begin{cases} 1 & m = \mu + M \le m_{\max} \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

This step function "masks" the galaxies in the upper triangle in Figure 1 so that they cannot be observed, simulating the effect of the limiting apparent magnitude m_{max}. Per our assumption that absolute magnitude is independent of position, we can express N_{true} as the product of the one-dimensional functions $\varphi(M)$ and $D(\mu)$:

$$N_{true}(M, \mu) = \varphi(M) D(\mu) \quad (23)$$

Our expression for N_{app} then becomes

$$N_{app}(M,\mu)=\varphi(M)D(\mu)\Theta\left[m_{max}-(M+\mu)\right] \tag{24}$$

Because the right-hand side of Eq. (20) is a sum of two-dimensional Dirac delta functions, and because $\varphi(M)$ and $D(\mu)$ are independent of one another, it is reasonable to express the right-hand side of Eq. (24) in terms of sums of one-dimensional Dirac delta functions:

$$\begin{aligned}\varphi(M)&=\sum_{i=1}^{N_{obs}}\psi_i\delta(M-M_i)\\ D(\mu)&=\sum_{j=1}^{N_{obs}}d_j\delta(\mu-\mu_j)\end{aligned} \tag{25}$$

Hence, our expression for $N_{true}(M, \mu)$ becomes

$$N_{true}(M,\mu)=\left[\sum_{i=1}^{N_{obs}}\psi_i\delta(M-M_i)\right]\left[\sum_{j=1}^{N_{obs}}d_j\delta(\mu-\mu_j)\right] \tag{26}$$

Combining Eqs. (20), (24), and (26) yields

$$\sum_{k=1}^{N_{obs}}\delta(M-M_k,\mu-\mu_k)=\left[\sum_{i=1}^{N_{obs}}\psi_i\delta(M-M_i)\right]$$

$$\left[\sum_{j=1}^{N_{obs}}d_j\delta(\mu-\mu_j)\right]\Theta\left[m_{max}-(M+\mu)\right] \tag{27}$$

Note that we can eliminate the mathematically awkward step function by judicious selection of the indices on the right-hand side of Eq. (27). This may be done by imposing the restriction that $M_i + \mu_j \le m_{max}$ for all i and j:

$$\sum_{k=1}^{N_{obs}}\delta(M-M_k,\mu-\mu_k)=$$

$$\sum_{i:}^{M_i+\mu_j\le m_{\max}}\sum_{j:}\psi_i d_j\delta(M-M_i)\delta(\mu-\mu_j) \tag{28}$$

Determining the Weighting Factors

In order to obtain a practical solution, it is necessary to solve for the ψ and d weighting factors on the right-hand side of Eq. (28). One may solve for a particular d_j value, say d_p, by integrating Eq. (28) over all values of M but only over one particular value of μ (Figure 4). To ensure that the integral of a Dirac delta function is nonzero, the argument of the delta function should fall *inside* the limits of integration (Griffiths, 1989, p. 49). Hence, in this particular case, one should technically start the integration just a little to the left of M_{min} and end it just a little to the right of M_{max}. Likewise, the second integration should be performed from $\mu_p - \varepsilon$ to $\mu_p + \varepsilon$, where ε is a vanishingly small number such that ε is just a little smaller than the minimum possible difference between any two adjacent values of μ_j. Having done so, one obtains an expression for each of the N_{obs} individual d_p values:

$$1=d_p\sum_{i:}^{M_i+\mu_p\le m_{\max}}\psi_i \quad p=1,2,\ldots,N_{obs} \tag{29}$$

Likewise, one can integrate both sides of Eq. (28) from μ_{min} to μ_{max} and from $M_q - \sigma$ to $M_q + \sigma$, where σ is a vanishingly small number such that it is smaller than the smallest possible difference between any two adjacent values of M_i. This yields, for a particular choice of $i = q$,

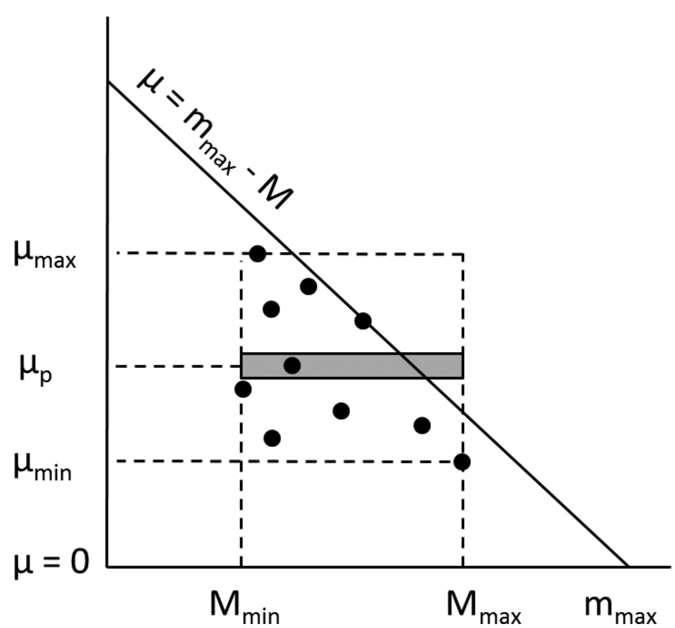

Figure 4. Area of integration used to obtain a specific weighting factor d_p.

$$1 = \psi_q \sum_{j:}^{M_q + \mu_j \le m_{\max}} d_j \quad q = 1, 2, \ldots, N_{obs} \tag{30}$$

Relating the Weighting Factors to the C_k's

However, we still need some way of relating these weighting factors to the observed number of galaxies within the survey. To this end, we redraw Figure 2, but without the small shaded rectangle centered on M_k (see Figure 5). We then define $C_k = C^{-}(M_k)$ as the number of galaxies contained within this large shaded rectangle, or more precisely, as the number of galaxies having values of M and μ such that $M_{min} \le M < M_k$ and $\mu_{min} \le \mu \le m_{max} - M_k$. Note that, since $M_1 = M_{min}$, zero galaxies satisfy the requirement that $M_{min} \le M < M_1$. Hence no galaxies are inside the box when $M = M_{min}$ and $C_1 = 0$. We may use Eq. (28) to derive a useful relation between the C_k's and the ψ_i values. We again integrate Eq. (28), but this time from $\mu = \mu_{min}$ to $\mu = \mu_{max}$ and from $M = M_{min} - \sigma$ to $M = M_k + \sigma$. Upon doing so, the left-hand side of Eq. (28) yields the visible number of galaxies such that $M \le M_k$ (remember that there are zero visible galaxies above the diagonal line). Note that this number of visible galaxies is equal to $C_k + 1$, since C_k excludes, by definition, the galaxy for which $M = M_{min}$.

The result, combined with integration of the right-hand side yields

$$C_k + 1 = \sum_{i:}^{M_i + \mu_j \le m_{\max}} \psi_i \int_{M_{\min} - \sigma}^{M_k + \sigma} \delta(M - M_i) dM$$

$$\sum_{j:}^{M_i + \mu_j \le m_{\max}} d_j \int_{\mu_{\min} - \varepsilon}^{\mu_{\max} + \varepsilon} \delta(\mu - \mu_j) d\mu$$

$$= \sum_{i=1}^{k} \psi_i \sum_{j:}^{M_k + \mu_j \le m_{\max}} d_j \tag{31}$$

Eqs. (30) and (31) may then be used to obtain an extremely useful recursion relation. If one substitutes Eq. (30) into Eq. (31) in order to eliminate the summation over j and also obtains an expression for C_{k+1} by replacing k in Eq. (31) with $k+1$, one can derive:

$$\psi_{k+1} = \psi_k \frac{C_k + 1}{C_{k+1}} \quad k = 1, 2, \ldots, N_{obs} \tag{32}$$

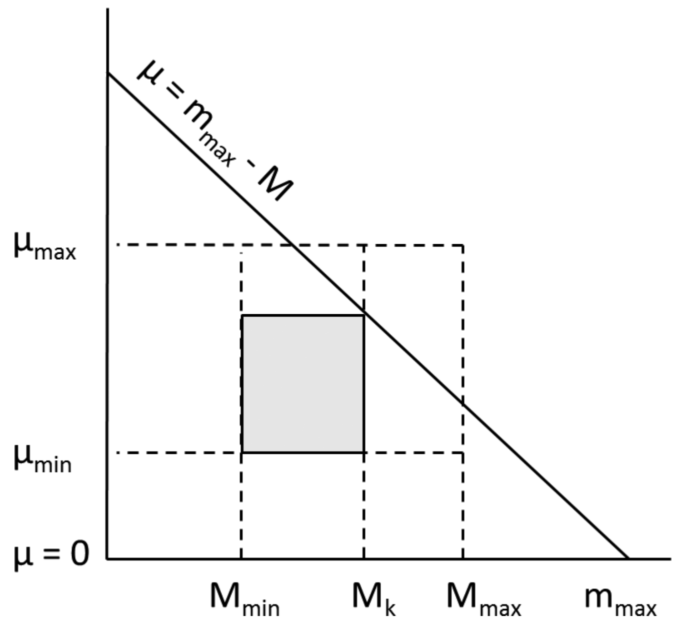

Figure 5. Geometry used to count the number of visible galaxies having absolute magnitudes between M_{min} and M_k. C(M_k) is defined to be the number of galaxies within the rectangle but excluding the galaxy on the rectangle's right-hand edge.

Since the C_k values may be obtained via simple counting, knowledge of ψ_1 automatically determines all the other values of ψ. Generally, ψ is taken to be equal to 1. However, the actual choice of ψ_1 is not critical, since the true density of galaxies in Eq. (28) for a given M_i and μ_j actually depends on the product of $\psi_i d_j$, not just ψ_i. Hence, choosing ψ_1 to be some other value than 1 will just result in different d_j values, leaving the overall factor $\psi_i d_j$ unchanged.

Obtaining the Comoving Volume and the Total Number of Galaxies

In order to express the luminosity function in terms of galaxies per unit volume, it is expedient to calculate the comoving volume of the survey. From our definition of the distance modulus in Eq. (3), we see that each value of μ (or equivalently, each value of z) corresponds to a unique luminosity distance d_L. Eq. (5) may then be used to convert this luminosity distance into a comoving distance. Once the minimum and maximum distance moduli (or the minimum and maximum z values) for the survey are used to obtain the minimum and maximum comoving distances $d_{c,min}$ and $d_{c,max}$, the appropriate total comoving volume for the survey may be obtained via

$$V_t = \int_{d_{c,\min}}^{d_{c,\max}} r^2 dr \int_{\text{solid angle}} d\Omega = \int_{d_{c,\min}}^{d_{c,\max}} r^2 dr \int_{\theta} \sin\theta' d\theta' \int_{\phi} d\phi' \tag{33}$$

It should be noted that Choloniewski seems to have made an error in his original paper. His originally defined V_t that he used to obtain the luminosity function (Choloniewski, 1987, p. 275) was actually expressed in terms of luminosity distance, rather than comoving distance, which is more appropriate for this kind of analysis. Upon obtaining V_t, our expression for the luminosity function becomes

$$\phi(M) = \frac{1}{V_t} \int_{\mu_{\min}-\varepsilon}^{\mu_{\max}+\varepsilon} N_{true}(M,\mu) d\mu = \frac{1}{V_t} \left[\sum_{i=1}^{N_{obs}} \psi_i \delta(M - M_i) \right] \left[\sum_{j=1}^{N_{obs}} d_j \right] \tag{34}$$

The total number of galaxies within the survey may be obtained by integrating our expression for N_{true}, Eq. (26), over all values of M and μ:

$$N_{total} = \int_{M_{\min}-\sigma}^{M_{\max}+\sigma} \int_{\mu_{\min}-\varepsilon}^{\mu_{\max}+\varepsilon} N_{true}(M,\mu) dM d\mu = \sum_{i=1}^{N_{obs}} \psi_i \sum_{j=1}^{N_{obs}} d_j \tag{35}$$

Obtaining the Luminosity Function

Once V_t and all the values of ψ and d have been obtained, the problem has, in principle, been solved. However, our solution is undefined for values of M that fall "between" the absolute magnitudes of the N_{obs} galaxies in Figures 1 and 4. Since we will need to integrate the luminosity function in order to obtain the selection function $S(z)$, it is expedient to obtain a version of the luminosity function that is defined at regularly spaced intervals of M. We denote this more useful version of the luminosity function as $\langle \phi(M) \rangle$, and it is obtained by integrating Eq. (34) over a small bin width of ΔM and then dividing the resulting numbers by ΔM:

$$\langle \phi(M) \rangle = \int_{M}^{M+\Delta M} \phi(M) dM \Big/ \int_{M}^{M+\Delta M} dM = \frac{\sum_{i:}^{M_i \in [M, M+\Delta M]} \psi_i \sum_{j=1}^{N_{obs}} d_j}{V_t \Delta M} \tag{36}$$

Once this version of the luminosity function has been obtained, it may be integrated, as in Eq. (6) to obtain the selection function $S(z)$. In order to obtain an estimate for $M_{lim(z)}$ for each value of z in the integration, it is necessary to insert m_{max} and $K_{\text{avg(z)}}$ into Eq. (17). $K_{\text{avg(z)}}$ can be obtained by averaging all the different K-corrections for the galaxies within a small bin of width Δz centered on that particular value of z.

Demonstration of the Method

Of course, there are additional complications that must be addressed when finding the luminosity function for a real galaxy survey. These are discussed in an accompanying paper (Hebert and Lisle, 2016), along with a demonstration of the LBC method.

Conclusion

The possibility that our galaxy may occupy a special location within the universe is obviously of great interest to the creation science community. However, testing of such a possibility requires the true (not just apparent) distribution of galaxies in the vicinity of our own Milky Way galaxy. This in turn requires determination of the luminosity function. Because methods used to obtain luminosity functions are rarely explained clearly in the technical literature, we have provided a detailed discussion of one such method. It is our hope that such an explanation will (1) enable the creation science community to intelligently critique claims that our galaxy occupies a privileged location in space and (2) serve as a "stepping stone" for future creation researchers.

References

Ball, N.M., and R.J. Brunner. 2010. Data mining and machine learning in astronomy. *International Journal of Modern Physics D*, Vol. 19, 7:1049–1106. http://ned.ipac.caltech.edu/level5/March11/Ball/Ball3.html#3.1

Bergström, L., and A. Goobar. 2008. *Cosmology and Particle Astrophysics*, 2nd edition. Praxis Publishing, Chichester, UK.

Chilingarian, I., A. Melchior, and I. Zolotukhin. 2010. Analytical approximations of K-corrections in optical and near-infrared bands. *Monthly Notices of the Royal Astronomical Society* 405:1409.

Choloniewski, J. 1987. On Lynden-Bell's method for the determination of the luminosity function. *Monthly Notices of the Royal Astronomical Society* 226:273–280.

Efstathiou, G., R.S. Ellis, and B.A. Peterson. 1988. Analysis of a complete galaxy redshift survey—II. The field-galaxy luminosity function. *Monthly Notices of the Royal Astronomical Society* 232:431–461.

Freedman, R.A., and W.J. Kaufmann III. 2002. *Universe: Stars and Galaxies*. W. H. Freeman and Co., New York, NY.

Gabrielli, A., F.S. Labini, M. Joyce, and L. Pietronero. 2010. *Statistical Physics for Cosmic Structures*. Springer-Verlag, Berlin and Heidelberg, Germany.

Griffiths, D.J. 1989. *Introduction to Electrodynamics*, 2nd edition. Prentice-Hall Inc., Englewood Cliffs, NJ.

Hartnett, J. 2005. A creationist cosmology in a galactocentric universe. *Journal of Creation* 19(1):73–81.

Hartnett, J. 2007. *Starlight, Time, and the New Physics*. Creation Book Publishers. Atlanta, GA.

Hebert, J., and J. Lisle. 2016. A review of the Lynden-Bell/Choloniewski method of obtaining galaxy luminosity functions: part II. *Creation Research Society Quarterly* 52:189–199.

Heydari-Malayeri, M. 2014. *An Etymological Dictionary of Astronomy and Astrophysics*. Observatoire de Paris. http://dictionary.obspm.fr/index.php/index.php?showAll=1&formSearchTextfield=Malmquist+bias (accessed August 11, 2014).

Hogg, D.W., I.K. Baldry, M.R. Blanton, and D.J. Eisenstein. 2002. The K correction. http://ned.ipac.caltech.edu/level5/Sept02/Hogg/paper.pdf (accessed May 2014).

Humphreys, D.R. 1994. *Starlight and Time*. Master Books, Green Forest, AR.

Humphreys, D.R. 2002. Our galaxy is the centre of the universe, 'quantized' redshifts show. *Journal of Creation* 16(2):95–104.

Lisle, J. 2016. New method to assess the luminosity function of galaxies. *Answers Research Journal* 9:67–79. https://assets.answersingenesis.org/doc/articles/pdf-versions/arj/v9/luminosity_function_galaxies.pdf

Lynden-Bell, D. 1971. A method of allowing for known observational selection in small samples applied to 3CR quasars. *Monthly Notices of the Royal Astronomical Society* 155:95–118.

Sparke, L.S., and J.S. Gallagher III. 2010. *Galaxies in the Universe: An Introduction*, 2nd edition. Cambridge University Press, Cambridge, UK.

Strauss, M.A., and J.A. Willick. 1995. The density and peculiar velocity fields of nearby galaxies. *Physics Reports* 261, Issues 5–6: 271–731. http://ned.ipac.caltech.edu/level5/March01/Strauss/Strauss_contents.html (accessed August 11, 2014).

Strauss, M.A., D.H. Weinberg, R.H. Lupton, V.K. Narayanan, and J. Annis, et al. 2002. Spectroscopic target selection in the Sloan Digital Sky Survey: the main galaxy sample. *The Astronomical Journal* 124:1810–1824.

Willmer, C.N.A. 1997. Estimating galaxy luminosity functions. *The Astronomical Journal* 114(3): 898–912.