As we study the various distributional patterns, we will probably learn that they represent special adaptations rather than an expression of transitional evolutionary stages.[5]

The basic developmental design for all vertebrate hearts involves a longitudinal tube which forms an S-shaped structure. Adult fish have this structure. For other vertebrates there is a folding of the tube and further development aided by hemodynamic conditions so that finally the organ characteristic of the species is formed.

Conclusion

In studies of the vertebrate heart it would appear beneficial if students of comparative anatomy could be relieved of the tension to explain details of cardiac anatomy and physiology in phylogenetic terms and be encouraged to understand the various features with primary attention on needs of particular organisms in their environments.

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AN ANALYSIS OF THE POST-FLOOD POPULATION GROWTH

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The world population growth after the Noachian Flood is analyzed. Several independent mathematical methods indicate that a population of many millions could have resulted after 200 years starting from Noah and his family.

Problem and Procedure

The population growth of the world population immediately after the Noachian Flood is examined both analytically and numerically. The author will ascertain whether or not geneological gaps are needed in Genesis 10 and 11 in order to account for a sufficiently large population capable of accomplishing the Tower of Babel, the Pyramids, and other Biblical and secular events, which must fall in the first few hundred years after the Flood, if one holds to a chronology after the manner of Ussher, Issac Newton, Courville, *et al.*

Computations show that, with respect to population growth, geneological gaps are not needed and that under reasonable assumptions, derived primarily from Genesis 10 and 11, a world population of many millions could have arisen within 200 years after the Flood. A review and some results of the following analytical approaches are presented:

(1) The problem has been formulted as an inhomogeneous birth-death stochastic process proceeding in the usual manner from the Chapman-Kolmogorov equation. A very large population growth after the Flood can be obtained but present day population statistics could not be superimposed on the model thus suggesting the extraordinary conditions in the immediate post-Flood era.

(2) Kendall's solution of the inhomogeneous birth-death process is discussed and a simple post-diluvian population formula is presented.

(3) Karlin's deterministic model is presented and a solution method suggested.

(4) The present state of the art of the formulation of such population problems is discussed.

In addition to analytical investigations, a direct numerical simulation for Karlin's model has been performed. These simulations and the other calculations were performed on The Cleveland State University IBM 370/158 system. This simulation conclusively shows the possibility of a large population growth as is verified and strengthened by analytical investigations.

Introduction

In recent years there has been a tendency, on the part of many creation scientists, to assume tacitly the existence of genealogical gaps in the geneologies of Genesis 5, 10 and 11 and elsewhere; for example see references 1-5. In fact, it is because of these alleged gaps that an age from the Creation of 10,000 years is being suggested more and more. This addition of nearly 4,000 years over the traditional chronologies of Ussher, I. Newton, Marsham, *et al.*,[6,7] has very possibly gained acceptance through the popular prolific writings of H. Morris and J. Whitcomb, who at least grant the possibility of gaps.

The writer holds that no gaps can exist; and that a complete and exact chronology is important to the Christian in order that he be watchful (e.g. Prov 8:34; Mt 24:42; Luke 12:37, 38; 21:36, Eph 6:18, 1 Pet 4:7; Rev 3:2, 3, 16:15) for signs and seasons (e.g. Mt 24:3, 30, Luke 21:11; Act 1:7; 1 Th 5:1; Heb 2:4; Rev 15:1), just as it was important to the wise men coming from the east (Mt 2:1, 2). By an exact chronology I mean a chronology to-the-day as providentially perfectly preserved in the King James Version (as *all* modern

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translations have taken liberty with numerical information thus obscuring the Biblical chronology).

It is not my purpose, here, to deal with the general subject of chronology and the attendant problem of Scriptural preservation, but instead to discuss only the problem of population growth immediately after the Flood. In this respect I concur with the sense of the recent post-Flood chronologies of Ozanne[8] and Courville.[9-12]

It is often implicitly alleged or stated that the lack of sufficient population immediately after the Flood necessitates the assumption of additional (missing) generations in Genesis 10 and 11. This assumption, one is told, is required due to the short time scale provided by the Ussher chronology versus the large population required by Biblical and secular events, such as the building of the Tower of Babel and the Pyramids. For example, it is frequently stated, that within Ussher's 200 years from the Flood it would be impossible to build the Tower with the small population available.

The Ussher chronology is not being defended specifically; however, the term will be used generically to represent chronologies incorporating no gaps of time, genealogical or otherwise (I hasten to add that Ussher's much maligned magnificent piece of research is indeed worthy of defense).

The principle being argued here is gaps versus no gaps; for if there must be gaps in Genesis 10 then why not in Genesis 5 or in the lineage of Moses (Ex 6), or David (Ruth 4), or any number of places bearing on the chronological reckoning of The Biblical record. This paper attempts to show that in Genesis 10, this most sensitive and argued place, one cannot argue for gaps based on insufficient population.

In the sections that follow, increasingly more general models for the post-Flood population problem will be presented. This problem is an age-dependent branching process having a variable longevity and reproductive period. This general problem is seldom treated in the literature, except qualitatively; and approximations seems to give only marginal insight and quantiative information. Several methods are included in order to provide a survey of mathematical tools applicable to Biblical population analysis and in order to provide a motivation for more general models.

(1) The H. M. Morris Solution

In reference 3 Morris provided the following hueristic solution based on the geometric series, from which he suggests that only 1000 people would be available to build the Tower of Babel if one accepts the Ussher chronology. Hence he argued for the possibility of gaps amounting to a few hundred years in Genesis 10 and 11.

Following Morris, let n_o be the initial population and c be the number of offspring per family. Assume that the total population continually remains equally male and female, and that no deaths occur; then it follows that

$$S_n = n_o + n_o c + n_o c^2 + \cdots = n_o(c^{n+1} - 1)(c-1)^{-1} \qquad (1)$$

where S_n is the total population after n generations. Morris corrects this sum by assuming all individuals have a life span of x generations,

$$S_{n,x} = S_n - S_{n-x} = n_o c^{n-x+1}(c^x - 1)(c-1)^{-1} \qquad (2)$$

On the basis of the Ussher chronology Morris seems to suggest $n_o = 8$, $n = 3$, $c = 5$, and $x = 5$ yielding $S_{n,x} = 1000$. This model rather well described the present day world population growth and provides a devasting refutation of evolution theory;[3] but it is less successful as a means to describe even moderately well the complexitities attendant with starting from a small number of individuals, 8, whose longevity is about 650 years as compared with 70 of their distant progeny.

The complication of a variable longevity and the associated variable reproductive period is inconsistent with the simple model provided by a geometric series. One cannot properly speak in terms of a constant generation as incorporated by n, and similarly for the longevity measure x and growth measure, c.[15]

These criticisms are quite obvious; still the simple geometric model has served well in stimulating further research in population dynamics.[13-15] Lastly, it should be noted that although this model is usually used to demonstrate a very small population at 200 years after the Flood, it likewise can be used to demonstrate a large population. For example if one neglects deaths, which seems to be permissible in light of the great life spans of man from Shem to Eber, then as an approximation $S_n/n_o = c^n$. If $n = 5$ and $c = 10, 15, 20, 25$ then $S_n/n_o = 10^5$, 7.6×10^5, 3.2×10^6, 9.8×10^6 respectively. It will be shown, subsequently, that more accurate versatile models also yield results in the millions or greater.

The Birth-Death Process

The Chapman-Kolmogorov equation is invoked in the usual manner[16,20] in order to model the problem as an inhomogeneous birth-death process. Let the process be defined by the transition probabilities for a single death, no change, one birth and a double birth where the following form is assumed;

$$\begin{aligned} p_2 &= P[x \to x+2, t\epsilon(t, t+\Delta t)] = c_2(t)\, x\Delta t \\ p_1 &= P[x \to x+1, t\epsilon(t, t+\Delta t)] = c_1(t)\, x\Delta t \\ p_0 &= P[x \to x, t\epsilon(t, t+\Delta t)] = 1 - (c_1 + c_2 + c_3) x\Delta t \\ p_{-1} &= P[x \to x-1, t\epsilon(t, t+\Delta t)] = c_3(t) x\Delta t \end{aligned} \qquad (3)$$

where x is the population at time t. c_1, c_2 and c_3 are functions of time only which, one hopes, can be selected so as to reflect the variable longevity of an individual. These probabilities have been assumed to be proportional to the current population. The Chapman-Kolmogorov equation for this process is

$$P(x, t+\Delta t) = \Sigma_{i=-1}^{2} p_i P(x-i, t) \qquad (4)$$

Combining and rearranging gives

$$\begin{aligned} P'(x,t) &= \lim_{\Delta t \to o} (P(x, t+\Delta t) - P(x,t))/\Delta t \\ &= c_2(x-2)P(x-2,t) + c_1(x-1)P(x-1,t) \\ &\quad -(c_1 + c_2 + c_3)xP(x,t) + c_3(x+1)P(x+1,t) \end{aligned} \qquad (5)$$

where a prime will indicate differentiation with respect to time. This differential-difference equation requires the following boundary conditions;

$$P(x_0, 0) = 1$$
$$P(x, 0) = 0, x \neq x_0 \quad (6)$$

where x_0 is the initial population at time $t = 0$. Let $m(t)$ denote the mean of this process,

$$m(t) = \Sigma_{x=x_0}^{\infty} xP(x, t) \quad (7)$$

from which after some summation manipulation in combination with the expression for $P'(x, t)$ gives,

$$m'(t) = \Sigma_{x=x_0}^{\infty} xP'(x, t)$$
$$\ldots$$
$$= R(t)\, m(t)$$
$$m(t) = m(0) \exp(\textstyle\int_0^t R(\tau)d\tau) \quad (8)$$

where,

$$R(t) = 2c_2 - c_1 - c_3,\ m(0) = x_0 \quad (9)$$

Consider the following selection for c_1, c_2 and c_3:

$$c_1(t) = a_1 e^{-\beta_1 t} + \delta_1 \qquad (a_1, \beta_1, \delta_1) > 0$$
$$c_2(t) = a_2 c_1(t) \qquad a_2 > 0$$
$$c_3(t) = a_3(1 - e^{-\beta_2 t}) + \delta_2 \qquad (a_3, \beta_2, \delta_2) > 0 \quad (10)$$

This selection is governed by its mathematical tractability, and its qualitative properties with regard to information in Genesis 10 and 11 and present day population statistics. Note that for large t the c_i approach positive constants, and the model becomes the simple-birth-death process[17] which satisfies population growth in present times.

The exponential contribution is included in order to acknowledge the peculiar circumstances of exponentially decreasing longevity after the Flood. This is a purely hueristic assumption for it is not clear that an exponential transition probability intensity should be able to express an exponentially time decaying longevity. A fortiori, the birth-death model is itself quite arbitrary, lacking sufficient flexibility to straightforwardly represent this complicated population problem.

Combining the c_i with the expression for $m(t)$ yields,

$$m(t) = m(o) \exp(V(t)t) \quad (11)$$

where

$$V(t) = A_0 + t^{-1}(B_1(1 - e^{-\beta_1 t}) + B_2(1 - e^{-\beta_2 t})) \quad (12)$$

and

$$A_0 = \delta_1(2a_2 + 1) - a_3 - \delta_2$$
$$B_1 = \beta_1^{-1} a_1(2a_2 + 1)$$
$$B_2 = \beta_1^{-1} a_3 \quad (13)$$

The function, $V(t)$ is the exponential growth parameter. For small t, the first two terms of the Taylor expansion for the exponential function gives, $V(t) = (A_0 + \beta_1 B_1 + \beta_2 B_2)t$. For large t, $V(T) = A_0$. This seems qualitatively correct in that immediately after the Flood the growth parameter increases with time whereas for large times the parameter is constant.

Let it be required to solve for A_0, B_1 and B_2 from three boundary conditions and given values for β_1 and β_2. Let $m(o) = 6$ be the generation surviving the Flood from which the world was repopulated. Let the present time be $t_p = 4300$ years and the present population be $m(t_p) = 2 \times 10^9$. (doubling this figure would not change the conclusions much.) Current world population figures of the relative rate of growth indicate that $m'(t_p)/m(t_p) = .018$.

Lastly, 15 male offspring of Noah's sons, Shem, Ham and Japeth are listed in Genesis 10. If there were an equal number of female offspring (Gen. 11:11) and, if t_1 is the length of this very first generation then $m(t_1) \approx 30$. It would seem reasonable to assume this number to be much larger. From Gen 11:10 it seems plausible, for the moment, to assign $t_1 = 10$. Next define,

$$k_1 = \ln(m(t_p)/m(o))$$
$$k_2 = m'(t_p)/m(t_p)$$
$$k_3 = \ln(m(t_1)/m(o)) \quad (14)$$

which with the just mentioned figures give, $k_1 = 20$, $k_2 = 0.018$ and $k_3 = 1.61$. Also, temporarily assign $\beta_1 = 10^{-4}$ and $\beta_2 = 10^{-4}$. Hence a linear system in A_0, B_1 and B_2 results:

$$A_0 t_p + B_1(1 - e^{-\beta_1 t_p}) + B_2(1 - e^{-\beta_2 t_p}) = k_1$$
$$A_0 + B_1\beta_1 e^{-\beta_1 t_p} + B_2\beta_2 e^{-\beta_2 t_p} = k_2$$
$$A_0 t_1 + B_1(1 - e^{-\beta_1 t_1}) + B_2(1 - e^{-\beta_2 t_1}) = k_3 \quad (15)$$

Due to the great size of t_p compared to t_1, even double precision computation (16 significant digits) fails to solve this system, i.e. a prohibitively small determinant results. Hence in order to test this model $m(t)$ vs. t has been computed for a wide selection of values for A_0, B_1, B_2, t_i, $m(t_1)$, β_1 and β_2 holding t_p, $m(t_p)$ and $m(o)$ as assigned. A great many plausible combinations yielded values of $m(200)$ on the order of 10^6 to 10^9. However, all these cases produced values of $m(t_p) \approx 10^{30}$ which are absurdly large compared to $m(t_p) = 2 \times 10^9$.

Hence, it seems that the birth-death model employing simple, but plausible, transition probabilities is capable of modeling either the immediate post-Flood population or modern population, but not both simultaneously. It especially is to be noted that very large early populations occur. No further attempt will be made to find expressions for the c_i that would fit the data embodied in the values of the k_i. One obvious embellishment would be to include a more prolific model for double births and to permit higher order multiple births. This will be discussed later.

(2) Kendall Inhomogeneous Birth-Death Process

Before proceeding to more complicated models it will be instructive to note the utility of Kendall's solution of the inhomogeneous birth-death process[16,17] in that it may provide useful population formulae for the post Flood period as well as other periods of Biblical history such as the exceptional growth of the Israelites in Goshen (Ex 1).

For the general problem given by

$$P[x \to x + 1, t\epsilon(t, t + \Delta t)] = \lambda(t)\, x\Delta t$$
$$P[x \to x - 1, t\epsilon(t, t + \Delta t)] = \mu(t)\, x\Delta t \quad (16)$$

Kendall[18,19] has solved for $P(x, t)$. This is here mentioned because it is about the only stochastic process of sufficient generality to be potentially applicable to Biblical research for which $P(x, t)$ can be explicitly exhibited.

From the previous section it follows immediately that

$$m(t) = m(o) \exp(\textstyle\int_0^t(\mu(\tau) - \mu(\tau))\, d\tau) \quad (17)$$

For example if

$$\lambda(t) = a_0 + a_1(t + a_2)^{-1}$$
$$\mu(t) = \beta_0 + \beta_1(t + \beta_2)^{-1} \quad (18)$$

for constants a_1 and β_1 then

$$m(t)/m(o) = (a_2 + t/a_2)^{a_1} (\beta_2 + t/\beta_2)^{\beta_1} e^{(a_0 - \beta_0)t} \quad (19)$$

These choices for $\lambda(t)$ and $\mu(t)$ permit rough approximations over $t \epsilon (0, \infty)$ for the exponentials of the last section[21] while yielding a simpler result.

This process and the fact that it can be analytically solved should be of special interest to Bible Science researchers. Arley[22] has used this process in presenting, probably, the most detailed mathematical study of cascade showers in cosmic ray theory. This is important in the anlaysis of C-14 dating.

Bailey[17] (p. 115) has generalized Kendall's solution for $P(x, t)$ to include the effects of immigration. Though this solution is quite complicated, it does provide a closed form solution which is susceptible to direct numerical evaluation or generating function methods. Detailed studies of the historical demography and population growth of the race of the messianic line cannot help but include the effects of immigration. For example, consider the Gibeonites (Josh 9) and the law of the stranger (Lev 19:34, Ex 22:21, Deut 31:12, Heb 13:2).

(3) Karlin Deterministic Model

In this section the rather general model of Karlin[23] (pg. 360) will be presented. Solution of the model by Monte Carlo simulation will be given in the next section. Simulation permits even more embellishments, complexities and generalities than do deterministic models. The important aspect of this deterministic model and of the subsequent simulation, is the inclusion of age-dependent branching and parental survival which is essentially a non-Markovian property, and hence may appear to be included into the Markovian birth-death process only by various contrivances.[16,17,23]

Let $\rho(a, t)$ be the frequency function of individuals of exactly age a in the population at time t, and $b(t)$ be the rate of new individuals being born at time t. Hence the fraction of the population between the ages a_1 and a_2 is $\int_{a_1}^{a_2} p(a, t)da$ and the number of new individuals created in time interval t_1 to t_2 is $\int_{t_1}^{t_2} b(t)dt$. Let $\lambda(a)dt$ be the expected number of progeny of a single individual of age a in dt units of time, $l(a)$ be the probability that an individual will survive from birth to at least age a, and $c(a)$ be the infinitesimal death rate (i.e. the probability of death of an individual of age a within the next h units of time $= c(a)h + 0(h)$). From the postulate, an individual will survive from birth $a + h$ units of time if and only if he survives from birth a units and then does not die in the next h units of time, it follows that

$$l(a + h) = l(a)(1 - c(a)h)$$
$$dl(a)/da = \lim_{h \to 0} (l(a + h) - l(a))/h = -l(a)\, c(a) \quad (20)$$

and hence

$$l(a) = l(o) \exp(-\int_0^a c(s)ds), \; l(o) = 1 \quad (21)$$

Therefore the problem becomes; given the functions λ and c(l following from c), determine b.

Let $b_0(t)$ be the rate of new individuals due to individuals in the population at time $t = 0$, and $b_1(t)$ be the rate due to individuals born at time $t > 0$, i.e.

$$b(t) = b_0(t) + b_1(t) \quad (22)$$

First $b_0(t)$ will be developed. Let P denote the conditional probability of survival to time t (i.e. to age $= a + t$) given an age of a at $t = 0$. Hence, $P \cdot$ prob(age $\geqslant$ a) = prob(age $\geqslant$ a + t), i.e.

$$P = l(a + t)/l(a) \quad (23)$$

Therefore the fraction of individuals of age a at $t = 0$ that survive to age $a + t$ (i.e. to time t) is

$$(l(a + t)/l(a))\, \rho(a, o) \quad (24)$$

and summing over all ages gives

$$b_0(t) = m(o) \int_0^\infty \lambda(t + s)\, (l(s + t)/l(s))\, \rho(s, o)ds \quad (25)$$

where $m(t)$ is the total population at time t.

Next $b_1(t)$ will be derived. The conditional probability of survival of an individual to time t (i.e. to age $= t - \tau$) given his age was 0 at time τ is $l(t - \tau)$, the rate of new individuals at time τ is $b(\tau)$, and the rate of births from individuals of age $t - \tau$ is $\lambda(t - \tau)d\tau$. Therefore the rate of new individuals at time t due to individuals born at $t > 0$ within the time interval $d\tau$ is

$$\lambda(t - \tau)\, b(\tau)\, l(t - \tau)d\tau \quad (26)$$

and on summing over all intervals of $d\tau$,

$$b_1(t) = \int_0^t \lambda(t - \tau)\, l(t - \tau)\, b(\tau)d\tau \quad (27)$$

Therefore combining results in an integral equation for $b(t)$,

$$b(t) = b_0(t) + \int_0^t \lambda(t - \tau)\, l(t - \tau)\, b(\tau)d\tau \quad (28)$$

where, as we have seen, $b_0(t)$ is a known function of t depending on the total initial population, $m(0)$, the initial age distribution, $\rho(a, o)$ and the functions λ and l(or c). The total population, $m(t)$ and the age distribution are related by

$$m(t)\, \rho(a, t) = b(t - a)\, l(a), \; a \leqslant t \quad (29)$$

A solution of the integral equation may be obtained numerically by standard procedures.[24,25] The following procedure converts the problem to a differential equation. Let ϕ denote λl and expand ϕ into its Taylor series,

$$b(t) = b_0(t) + \int_0^t \phi(t - \tau)\, b(\tau)d\tau \quad (30)$$
$$= b_0(t) + \int_0^t (a_0 + a_1(t - \tau) + a_2(t - \tau)^2/2! + \cdots)b(\tau)d\tau$$

where

$$ai = (d^i \phi(s) / ds^i)_{s = 0}; \; i = 0, 1, 2, \cdots, n \quad (31)$$

Using the well known integral identity $b(t)$ becomes,

$$b(t) = b_0(t) + a_0 \int_0^t b(\tau)\, d\tau + a_1 \int_0^t \int_0^t b(\tau)\, d\tau^2$$
$$+ \cdots + a_n \int_0^t \cdots \int_0^t b(\tau)\, d\tau^n \quad (32)$$

and differentiating n time gives,

$$b^{(n)} - a_0 b^{(n-1)} - a_1 b^{(n-2)} - \cdots - a_n b = b_0^{(n)} \quad (33)$$

This is an inhomogeneous differential equation with constant coefficients. Initial conditions are provided by the values of $b_0(t)$ and its first $n - 1$ derivatives evaluated at $t = 0$. This equation can be solved analytically where an appropriate approximation of the particular solution is obtained for example, by evaluating,

$$\prod_{i=1}^{n} (D - r_i)^{-1} b_0^{(n)}(t) \qquad (34)$$

where r_i are the characteristic roots of the differential equation and D is the differential operator $D = d/dt$. The author has developed analytical computer aided techniques that employ automatic algebraic and symbolic manipulation for solving such problems.[26] This equation could be numerically solved by traditional methods such as Runge-Kutta integration.[27] The accuracy of the solution increases with increased n but so does the labor of obtaining it.

(4) Simulation Solution

In this section $b(t)$, and consequently, $\rho(u, t)$ and $m(t)$, will be obtained by a computer simulation. These computations, as well as the above, have been performed on the Cleveland State University IBM 370/158 system. The solution by simulation permits the easy inclusion of features that would present enormous difficulty in regard to analytical or traditional numerical methods, for example, the selection of very complicated $\lambda(t)$ and $c(t)$ which may be discontinuous. It should be noted that the accuracy of a solution by simulation is not subject to round-off error nor error of truncation but, instead, limited by the variance of the simulation.[28]

Let $f(u, t)$, be the frequency histogram, at time t, of individuals born in birth quantile u, $(u = 1, 2, ...)$. Hence the age of an individual is $a = t - u$ and $\rho(a, t)$ is the normalized continuous representation of the discrete $f(t - a, t)$. It will be convenient to solve for $f(u, t)$ directly instead of obtaining $b(t)$ from which $\rho(a, t)$ follows. The initial distribution $f(u, o)$ must be given. Instead of using $b(t)$, $\lambda(a)$, $l(a)$ it will be more convenient to use the related functions $\phi_b(a, u)$ and $\phi_d(a, u)$, which are, respectively, the probability of birth (possibly multiple) from an individual of age a and born in birth quantile U, and the probability of death of an individual of age a who was born in birth quantile u. Let $w_i(a, u)$ be the probabilities of i offspring given that a birth occurred, and require that

$$\Sigma_i w_i(a, u) = 1 \qquad (35)$$

The following correspondences with the continuous functions of the previous section result:

$$\lambda(a) \leftarrow \Sigma_i i w_i(a, u) \equiv S(a, u)$$
$$b(t) \leftarrow f(a, t)\, \phi_d(a, u)\, S(a, u)$$
$$l(a) \leftarrow 1 - \phi_d(a, u) \qquad (36)$$

The specification of $f(u, o)$, $w_i(a, o)$, $\phi_b(a, u)$ and $\phi_d(a, u)$ completely specify the simulation. Some of the details of the simulation are indicated by the flowcharts in Figure 1. These computations develop the 2-dimensional histogram, $f(u, t)$, by calling upon the procedures BIRTH and DEATH, which, respectively, simulate births and deaths by returning a correction, Δf, to the current value of $f(u, t)$. The final 2-dimensional histogram for $f(u, t)$ is obtained by averaging many applications of procedure POPULATION.

The simulation just described has been applied to the *only* data on the initial and early peopling of the world, i.e. the data of Genesis 10 and 11. Let time $t = 0$ represent the birth of Shem, Ham and Japheth and their wives, all of which, for simplicity, will be assigned to birth quantile $u = 1$ at $t = 0$ at 100 years before the Flood (Gen 5:32, Gen 7:6-11; Gen 8:13, Gen 10:1, Gen 11:10).

This assumption is probably close to fact; and various other distributions of the original six produce similar $f(u, t)$ from the Flood onward in time. Since no mention is made of additional offspring of Noah and his wife, they are not included in the original population. These arguments suggest

$$f(u, o) = \begin{cases} 6, & u = 1 \\ 0, & u > 1 \end{cases} \qquad (37)$$

In order to include the effect of the very large longevity that decreases continuously and rapidly to a limiting life span of 70 years (Ps 90:10, David's 70 years 2 Sam 5:4, 1Kg 2:10), let $T(u)$ denote the life span of the post-Flood population and let

$$\phi_d(a, u) = \begin{cases} 0, & 0 \leqslant a < a_1 \\ a/T, & a_1 \leqslant a < T \\ 1, & T \leqslant a \end{cases}$$

$$\phi_b(a, u) = \begin{cases} 0, & 0 \leqslant a < 20 \\ c_b, & 20 \leqslant a < .5T \\ 0, & .5T \leqslant a \end{cases}$$

$$T(u) = 70 + 600\, e^{-u/200} \qquad (38)$$

$T(u)$ is an exponential function whose form attempts to fit the longevity data of Genesis 11.[29] $T(u)$ depends only on the time of birth of the individual as might be expected when considering the debilitating effect on developing organisms due to the increasingly harsh global environment after the Flood.[1]

Rapidly increasing ultraviolet radiation and the decreasing terrestrial magnetic field are two possible reasons for this debilitation. Reference 30 is especially significant in associating longevity solely with the time of birth. The choice of $\phi_d(a, u)$ reflects the increased probability of death as time increases while the selection for $\phi_b(a, u)$ attempts to incorporate the shortening reproductive period.

It is convenient to make no distinction between male and female; or else to take three as the initial population and choose a male-female pair as the individual. These two treatments are conceptually and mathematical equivalent; the latter case yielding half the actual total population. The reproductive period has been set at between 20 years of age; and half the individual's life span during which time a birth occurs with constant probability c_b.

The most sensitive parameter of this simulation is clearly c_b. If c_b is assigned $c_b = 0.1$ then this may be understood as there being, on the average, one birth (possibly multiple) per each 10 year period (within the reproductive period) per individual. The probabilities of multiple births will be taken as constant, specifically,

$$w_1 = 1, w_2 = w_3 = \cdots = 0 \qquad (39)$$

The value a_1 will be conservatively assigned as zero in all computations.

Table 1 shows the result of the simulation for the above definitions with $c_b = 0.5$. Similar numbers result for $c_b \geqslant 0.2$. For $c_b = 0.1$ the population dies out but for $c_b = 0.15$ the population grows to 2.1×10^6 by 200 years after the Flood.

A time argument value of $t = 300$ corresponds to 200 years after the Flood, the Flood having been assumed to occur 100 years after the beginning of the simulation. During this time the world was greatly and quickly repopulated while concurrently losing its ability to do so.

Only a general population growth model, such as this one, could accommodate these conflicting developments. The total population of $m(300) = 6.2 \times 10^7$ strongly indicates that the world could, indeed, be greatly repopulated by 200 years after the Flood and that, consequently, geneological gaps need not be assumed in the record of Genesis 10 and 11, at least, on the basis of otherwise insufficient world population. Figure 2 shows the variation of $m(300)$ and $m'(300)/m(300)$ versus c_b.

Clearly, $m(300)$ strongly depends on c_b. Before embarking on arguments for assigning the value of c_b, let it be noted that changes in the forms of $T(u)$, ϕ_b and ϕ_d do not sensibly change the order of magnitude of the outcome of these numerical experiments. In fact, most plausible changes would increase $m(300)$.

For example, increasing a_1 and decreasing w_1 (i.e. increasing the w_i, $i > 1$) might increase $m(300)$ by a factor of 10 if a fairly high frequency of multiple births is permitted. Changing the three constants in the expression for $T(u)$ throughout a large range of values has little effect on the outcome. The constants in the expression for ϕ_b (i.e. the 20 and 0.5) have an observable effect on $m(300)$, but an effect that is smaller than the sensitivity of the outcome on c_b.

However, the dependence on the width of the reproductive period would be similar to that on c_b. A PL/1 program listing of the simulation in Figure 1 for the expressions of the text (but using two years and a male-female pair as units) is available from the author.

Various other plausible functional forms for ϕ_b and ϕ_d were found to yield similar results.

A value of $c_b = 0.3$ and $w_1 = 1$ is equivalent to postulating a single-birth within every 3.33 year period within the reproductive period. This has been approximately the case for overpopulated areas at the present time. Large infant mortality has suppressed the recent world population growth. One need not assume such infant mortality in the immediate post Flood era.

Furthermore, at the present time many ethnic groups such as the Hutterites of Canada, the Old Order Amish of Ohio, and the French Canadians of Quebec, to name a few, nearly possess this reproductive rate. It has been suggested that the main contributing factor for the successful conquests of the Vikings was a population explosion. This may have been commensurable to assigning $c_b = 0.3$. Many other historical examples can be cited. A conservative value of $c_b = 0.175$ (a birth every 5.71 years) yields a sufficiently large $m(300)$ to demonstrate the thesis of this paper.

If a high frequency of multiple births is permitted then a vastly higher population growth occurs. If $c_b = 0.3$, $w_1 = 0.9$ and $w_2 = 0.1$ then, roughly speaking, an effective c_b, $\overline{c}_b$, of 0.3 $(1 \times 0.9 + 2 \times 0.1) = 0.33$ results. Even an increase in c_b of 10% stimulates population growth by about a factor of about 1.12. It is easy to see that if twins, triplets and even higher multiples are permitted then it is not unreasonable to expect $\overline{c}_b = 1.5c_b$ which, by Table 1, corresponds to an increase in population by a factor of about 1.7. Increase of the width of the reproductive period would similarly effect c_b.

It is interesting to speculate on whether or not the Biblical record of the post Flood period or non-Biblical sources for this time support a high frequency of multiple births, i.e. is the ratio $\overline{c}_b/c_b$ substantially greater than 1.

The geneological information from Genesis 10 to the end of Genesis includes about 250 names (Gen 10, 11, 22, 25, 36, 46, 50) which comprises only a partial list

Table 1. The population distribution every 20th year since the Flood grouped into 20 year intervals of birth quantiles. Column 1 contains the average number of first parents. Columns 12 and 13 contains the total population, m, and the relative fractional population growth. $c_b = 0.5$, $w_1 = 1$.

		20 year intervals of birth quantiles, u.										
	1	101-120	121-140	141-160	161-180	181-200	201-220	221-240	241-260	261-280	281-300	m	m'/m
0	6.00	0	0	0	0	0	0	0	0	0	0	$6.0 \cdot 10^0$	—
20	6.00	$3.0 \cdot 10^0$	0	0	0	0	0	0	0	0	0	$2.4 \cdot 10^2$	$5.0 \cdot 10^{-2}$
40	3.82	3.0	$1.8 \cdot 10^4$	0	0	0	0	0	0	0	0	$1.8 \cdot 10^4$	5.0
60	1.20	3.0	1.8	$8.8 \cdot 10^5$	0	0	0	0	0	0	0	$1.0 \cdot 10^6$	5.0
80	0.26	3.0	1.8	8.8	$8.6 \cdot 10^6$	0	0	0	0	0	0	8.4	4.4
100	0	3.0	1.8	8.8	8.6	$8.4 \cdot 10^6$	0	0	0	0	0	$1.8 \cdot 10^7$	2.8
120	0	3.0	1.8	8.8	8.6	8.4	$1.1 \cdot 10^7$	0	0	0	0	2.9	1.5
140	0	2.2	1.8	8.8	8.6	8.4	1.1	$9.0 \cdot 10^6$	0	0	0	3.9	1.3
160	0	1.1	1.4	7.0	8.6	8.4	1.1	9.0	$1.1 \cdot 10^7$	0	0	4.7	$8.5 \cdot 10^{-3}$
180	0	$2.4 \cdot 10^1$	$4.6 \cdot 10^3$	5.2	7.6	8.4	1.1	9.0	1.1	$9.2 \cdot 10^6$	0	5.4	7.4
200	0	$3.8 \cdot 10^0$	1.1	1.5	4.4	8.4	1.1	9.0	1.1	9.2	$8.5 \cdot 10^6$	6.2	6.8
	1	2	3	4	5	6	7	8	9	10	11	12	13

of, essentially, only male offsrping. Of these names only Esau and Jacob (Gen 25:24), and Pharez and Zarah (Gen 38:27) are stated to be twins, which would indicate only a $w_2 = 2/246 = 0.008$. A detailed search of the geneological tables of Genesis would suggest several cases which might be at least twins, e.g. Peleg and Joktan (Gen 10:25), and Buz and Huz (Gen 11:21). On this basis one might conjecture a $w_2 = 0.1$.

An even larger w_2, and even significant values for w_3 through w_6, is suggested by the Talmud. Herein the commentary on Ex 1:7 holds that sextuplets were common and observes that the word for "multiplied" pertains to reptilian reproduction whereby multiple births were the rule. In this case a large $\bar{c}_b/c_b$ would not be difficult to substantiate.

The King James Version indicates this distinction, implicit in the Masoretic text, by using the word "fill" in Ex 1:7, as in Gen 1:22 in distinction to "replenish" in Gen 1:28 and Gen 9:1. The word "fill" meant to stock a volume (i.e. the sea and air) with animals, whereas the word "replenish" specifically is used to indicate stocking a land area with humans.[31]

It is not clear as to whether or not such fecundity can be imputed to previous generations; for, at least, it is clear that the descendants of Mizr (the Egyptians) were not as prolific as the descendant of Eber (the Hebrews). Some have even argued for the possibility of Cain and Abel being twins.[32]

The argument for a large population growth after the Flood does not, however, depend on frequent multiple births. This would only strengthen the argument; as would other assumptions such as assigning a non-zero value to a_1, including Noah and his wife in the initial population, or increasing the duration of the reproductive period to beyond half the life span. Though it be supernatural, the case of Sarah (Gen 17:7, 21:5) would suggest substantially increasing the upper limit from $0.5T$. Sarah's outstanding beauty, even in her old age (Gen 20), may further indicate how long the prime of life lasted.

The Abrahamic Promise, though it primarily applies to events far removed from Abraham's time, might be taken to indicate the especially rapid population growth in those days, in that the number of descendants of Abraham are likened to the "dust of the earth" (Gen 13:16), the number of "the stars" (Gen 15:5), and "the sand which is upon the sea-shore" (Gen 22:17).

A few comments on the arithmetic of the simulation are required. The random number generator used has been the well-tested power-residue generator.[28,32] A population simulation is a simulation on the positive integers; however, in the computations of Figure 1 fractional numbers result. Two procedures have been tested: retaining fractional numbers in that they represent the average of many such numerical experiments, or always rounding off to the nearest integer. Tests indicate no substantial difference between these two modes.

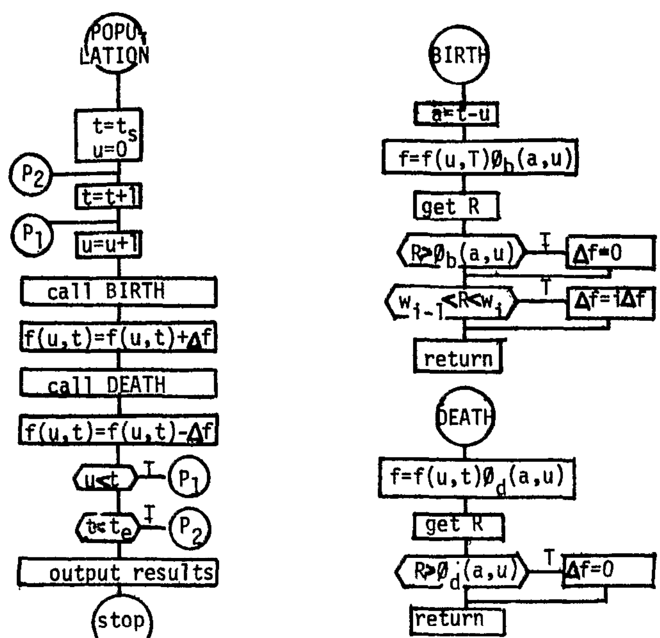

Figure 1. A brief flowchart indicating the simulaion of $f(u, t)$. t_e is some terminal value at which the simulation is terminated. R is a uniformly distributed random number between 0 and 1 which has been provided by the usual residue calculation.[28] $w_0 = 0$. t_s is the time at which population growth commences, which has been assigned the value of 100 as argued in the text.

Population Mathematics and Biblical Research

A number of mathematical tools are potentially useful for demographic and population studies associated with Bible research. A few have been detailed here, i.e. Kendall's solution of the inhomogeneous birth-death process, Karlin's deterministic model with the solution method suggested herein, and the use of simulation. More detailed studies should not exclude the work of Harris, Galton or Bellman (see bibliographies of references 16, 17, 23), or the application of the Volterra population equation.[34-36] The state of the art of the theory of age-dependent branching models is represented by the work of Savits.[37-39] Savit's theory generalizes the Bellman-Harris model to include all the complexities required by the problem of this paper;[37] however much research will be required in order to solve Savit's equations.

Similar anlayses could be applied to the growth of the ancient Assyrians, Babylonians and Persians, for example. This would be of value to secular, as well as Biblical, research. The author knows of no such attempts. It seems that secular historians, anthropologists and archeologists have used little quantitative analysis. This is not surprising, since they disregard or make light of the Bible as a source of data, thus ignoring the only record of the most ancient times and of beginnings, which, in fact, is an infallible inerrant record, and which uniquely provides much census and demographic information over such a long span of history, e.g. Gen 10:11, Num 1-3, 31-34, Josh 15-19, 2 Sam 24, 1 Chron, Ezra 2, Neh 7, and Est 9.

In addition to analyzing the population growth after the Flood it would seem worth while, likewise, to study the demographics of this population as it spread forth from Mt. Ararat (Gen 8:4) with discontinuity at Shinar (Gen 11:2). Very little quantitative research in this area has been done although extensive qualitative [40-42] and some semi-quantitative [43] is available. References 44-46 are worthy of mention.

Conclusion

The curves in Figure 2 may be regarded as lower bounds for a given c_b. Hence values of $c_b \geqslant 0.175$ (roughly, a single birth within every 5.71 year interval of the reproductive period) result in values of $m(300)$ (i.e. the world population 200 years after the Flood) from 10^7 to 10^8, provided the assumptions of the simulation model are sufficiently correct. Similar results have been obtained from analytic studies, thus verifying and strengthening the final conclusion: that a population on the order of millions *could* result shortly (e.g. 200 years) after the Flood. As a corollary it may be stated that, based on the data of Genesis 10 and 11, such population growth *did* result.

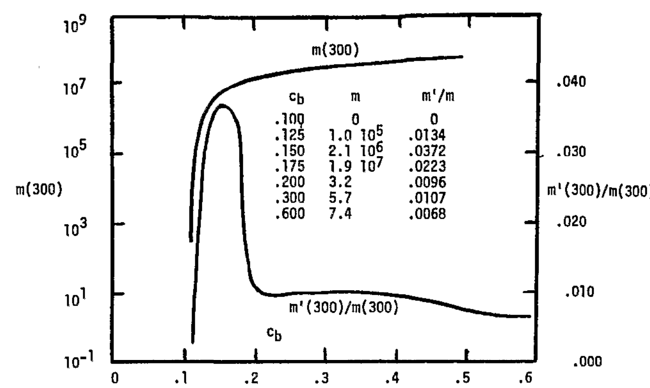

Figure 2. The world population, $m(300)$ and fractional population growth, $m'(300)/m(300)$, 200 years after the Flood, under the assumptions and assigned values of the text.

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