THE FARADAY-DISC DYNAMO AND GEOMAGNETISM

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The Faraday-disc dynamo has been put forward as a model to explain the variation with time of the earth's magnetic field. Evolutionists, especially, have claimed that this model explains the reversals of the field, which are believed to have occurred in the past; and that the model is consistent with the evolutionary doctrine. In this article the model is investigated in detail. The conclusion is that the magnetic field generated by the Faraday-disc dynamo never reverses; rather it decays exponentially. This exponential decay of the earth's field is a feature of the theory of geomagnetism proposed by Barnes. That theory offers one of the strongest proofs of a recent Creation. Thus the evolutionists' model is actually evidence, not for uniformitarianism, but rather for a recent Creation.

I. Introduction

There are two opposing explanations for the origin of the earth's magnetic field. The explanation consistent with the vast ages of evolution is the dynamo theory.[1-3] The explanation consistent with a recent creation is the Barnes theory.[4-8]

In both theories, the earth is believed to have a molten electrically conducting core (although the Barnes theory would not require that it be molten). Electrical currents are thought to flow in this conducting core. The earth's core is essentially a giant electromagnet. From this point on, the Barnes theory and the dynamo theory offer different explanations for two magnetic phenomena: 1) the slow decrease of the strength of the earth's magnetic field, and 2) reversed magnetized rocks.

A. Decreasing Geomagnetic Field: Barnes Theory

In the Barnes theory, the electrical currents in the earth's core all circulate in the same direction. These currents decrease with time, because of resistance leading to energy loss via electrical heating. Barnes showed that the measured rate of decay of the earth's magnetic field results from his model when a reasonable value for the electrical conductivity of the earth's core is used.[9] It is a major triumph for the Barnes model that its numerical predictions for the decay of the earth's magnetic field match the measured field better than any other theory.[10]

According to the Barnes theory, the earth's magnetic field must have been much greater in the past in order to decay down to its present value. The half life of the decay is approximately 1,400 years. At that rate only 50,000 years ago, the magnetic field of the earth would have exceeded the strongest magnetic fields known in the universe. The electrical heating in the earth's core would be 10,000 times the heat produced by the sun now. The earth would have exploded instantly, if it had ever been in such a condition. Thus, the Barnes theory shows that the earth could not have been in existence more than about 10,000 years, at the most.

The Barnes theory rests on the solid ground of electrodynamics. Lamb[11,12] and Barnes,[13] obtained an exact solution for the rigorous equations that describe electrodynamics, Maxwell's equations. So far, the Barnes theory offers everything one could want from an explanation. It has an exact theoretical foundation, and it has the best agreement with the experimental data.

The magnetic field of the Barnes model has never reversed direction, although it does decay. If there ever did come a time at which the earth's magnetic field tried to reverse drection, the field would have to pass through zero magnetic field for a while. The only way a zero magnetic field would exist is if the core currents were zero. However, once the core currents were zero, they would remain zero forever, because there is nothing to start them up again in the reversed direction. A current flow cannot just begin itself. Therefore, if the Barnes model is correct, the earth's magnetic field has not reversed itself in the past at any time.

The only evidence not in agreement with the Barnes model is reverse magnetized rock. If the earth's magnetic field has never changed directions in the past, how could these rocks have become reversed magnetized? The Barnes model in itself contains no explanation. One can suggest possible causes of anomalous magnetism; for example, a rock could be reversed magnetized by a lightning strike. However such happenings would not be an integral part of the Barnes theory.

B. Reversed Magnetized Rock: The Dynamo Theory

Almost all rock that contains magnetic material is found oriented with the magnetization of the magnetic material pointing in the general direction of the earth's north pole. It is usually assumed (without proof) that these rocks used to be molten, and that their magnetization lined up with the direction of the earth's magnetic field at the time they cooled below the Curie temperature.[14] The Curie temperature is the temperature at which the magnetism in a magnetic material essentially freezes. There are however, regions, roughly parallel to the longitude lines on either side of the mid-Atlantic ridge, in which the magnetization in rocks is found to point toward the earth's south pole.

Advocates of the dynamo theory cling to these rocks. They insist that the existence of reversed magnetized rocks is sufficient proof that the earth's magnetic field has reversed itself many times in the past. How these postulated field reversals could have occurred has never been explained theoretically. There has been much theoretical work spent on the dynamo model; but so far, no theory has been adequate to explain field reversals.[15]

The dynamo theory alleges that the electric currents in the earth's core somehow self-energize themselves and change themselves. Every million years or so the

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currents will interact with themselves so that the direction of the magnetic field they produce is reversed. Then, they supply themselves with energy necessary to maintain themselves over the next million years. Then the currents interact with themselves again so as to reverse the magnetic field another time. The cycle repeats itself again and again throughout the alleged evolutionary age of the earth.

Slowly, over the millions of years of geologic time, molten material is thought to emerge from the mid-Atlantic ridge. The material solidifies with the rock magnetism pointing toward whichever pole happens to be the magnetic north pole during that million year period, say pointing south. Then the magnetic poles reverse. During the next million years, fresh magnetic material is forced from the mid-Atlantic ridge, and the old material is pushed away from the ridge. This emergence of fresh material and sea floor spreading of old material as the magnetic field reverses each million years is supposed to account for the magnetically striped regions in the floor of the Atlantic Ocean. These regions consist of narrow stripes stretching almost from pole to pole. One stripe has rocks magnetized pointing north. The adjacent stripes have rocks magnetized south.

C. Choice of Theories: The Faraday Disk Dynamo

How did the dynamo theory with its non-conservation of energy, violation of the laws of electrodynamics, and lack of theoretical foundation, ever gain the almost universal acceptance of interested scientists? Why has Barnes' model with its solid theoretical foundation and excellent agreement between theory and data been ignored?

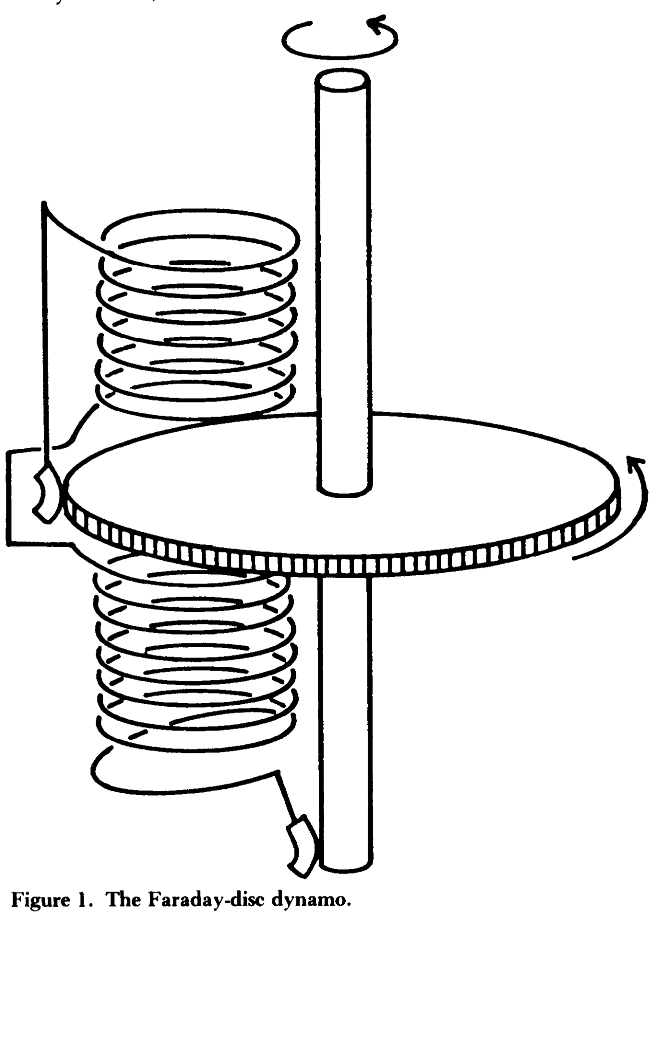

Figure 1. The Faraday-disc dynamo.

Dynamo theorists have a most persuasive, over-used example that they claim demonstrates the possibility of field reversals. Their example is the Faraday disk dynamo. The Faraday disk dynamo as built for e.g. lab. demonstration, is the arrangement shown in Fig. 1. Part of its appeal is its simplicity. Dynamo theorists claim that just as the current in the rotating disk produces a magnetic field which in turn induces a current in the disk, so can molten material in the earth act. The current and the magnetic field interact with one another and support one another. Dynamo theorists claim that this coupled interaction allows field reversals. They claim theoretical support from the fact that the equations governing the motion involve the square of the magnitude of the magnetic field, not the first power. Thus, they say, the field could reverse directions and still be described by the same equations.

This article examines the Faraday disk dynamo theoretically using well-known laws of electrodynamics. The results of this research are noteworthy. As stated in the abstract, the Faraday disk dynamo actually produces a *decaying* magnetic field rather than a reversing one. The interaction between the magnetic field and the currents can cause the rotating disk to reverse rotational directions once at most. Whether or not the disk undergoes its one allowed rotational reversal, the magnetic field *never* reverses. Thus, the Faraday disk dynamo, grasped so tightly by evolutionists, ironically supports the recent creation model of Barnes. The remainder of this article is devoted to a rigorous mathematical analysis of the electrodynamics governing the Faraday disk dynamo. This analysis is the only way to prove or disprove the alleged field reversals.

II. Equations of Motion

The Faraday disk dynamo (which can be built as a man-made device) is illustrated in Figure 1. The motion of the rotating conducting disk through the magnetic field B induces a current i according to Faraday's law. The electric current is used in an electromagnet to generate the magnetic field B according to Ampere's law. The magnetic field coupled to the rotating disk constitutes the simple Faraday disk dynamo.

The following symbols refer to the Faraday disk dynamo in Figure 1. $i = i(t) =$ the instantaneous current flowing through the disk and electromagnet; $R =$ the total resistance of the electrical circuit composed of disk, electromagnet, external wires, and contracts; $L =$ the inductance of the electromagnet; $n =$ the number of turns per unit length in the electromagnet; $r =$ the radius of the disk; $\mu_o =$ the permitivity of free space; $\tau =$ the torque on the disk due to electromagnetic effects; $I =$ the moment of inertia of the disk; $\epsilon_I =$ the E.M.F. generated in the portion of the disk passing through the magnetic field; $\epsilon_L =$ the self-inductance E.M.F. of the electromagnetic; and $\omega =$ the angular velocity of the disk ($\omega = 2\pi/\tau$).

The Faraday disk dynamo is equivalent electrically to

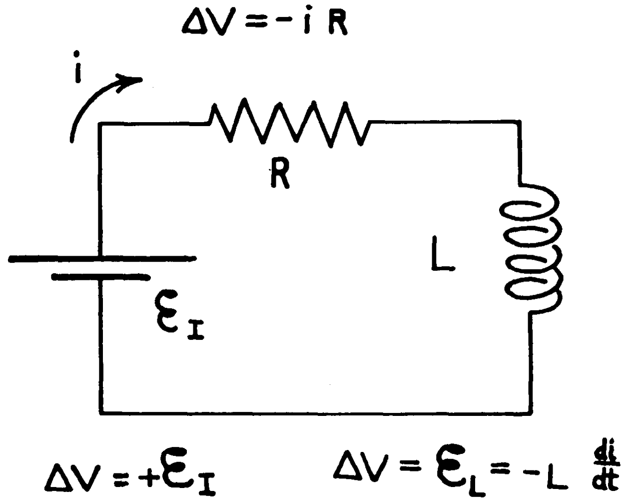

Figure 2. The electrical circuit equivalent to the Faraday-disc dynamo of Figure 1.

the electrical circuit shown in Figure 2. The current flow in the circuit is determined by Kirchhoff's voltage law,

$$\epsilon_I - R + \epsilon_L = 0. \quad (1)$$

Faraday's law gives

$$|\epsilon_I| = |d\Phi_B/dt| = B|dA/dt| = B \cdot {}^1\!/\!_2 r\omega r,$$

or

$$\epsilon_I = {}^1\!/\!_2 r^2 \omega B. \quad (2)$$

Ampere's law gives

$$B = \mu_o n i. \quad (3)$$

The self-inductance *L* is defined by the equation

$$\epsilon_L = -L\, di/dt. \quad (4)$$

When equations (2), (3), and (4) are substituted into the circuit equation (1), the current *i* is determined by the relation

$$({}^1\!/\!_2 r^2 \omega \mu_o n - R)\, i - L \frac{di}{dt} = 0. \quad (5a)$$

Multiplying this equation by *i* yields

$$({}^1\!/\!_2 r^2 \mu_o n \omega - R)\,(i^2) - \frac{L}{2}\frac{d(i^2)}{dt} = 0. \quad (5b)$$

The solution to equation (5a) is not a simple exponential, because the coefficient of the term linear in *i* contains the angular velocity ω of the wheel, and ω can vary with time.

The variation of ω is determined by the laws of rotational mechanics. The motion of the current-carrying conductor through the magnetic field results in a retarding torque on the disk. The retarding force ΔF on a thin strip of the disk through which the current flows between radii ℓ and $\ell + \Delta\ell$ is $|\Delta F| = iB\Delta\ell$. Therefore, the retarding torque on the thin strip is $|\Delta\tau| = iB\int_0^r \ell d\ell = {}^1\!/\!_2 iBr^2$. When the expression for B in equation (3) is used,

$$|\tau| = {}^1\!/\!_2 \mu_o n r^2 i^2. \quad (6)$$

Since this torque is a retarding torque, the motion of the disk is given by $-|\tau| = I d\omega/dt$, or:

$$i^2 = -\frac{2I}{\mu_o n r^2}\frac{d\omega}{dt}. \quad (7)$$

III. Constant Angular Velocity Solution

If one turns the disk (in a model as made for laboratory demonstrations) by hand in a way such that he forces ω to be a constant, the current flow can be determined from equation (5a) alone. However, three cases must be considered, corresponding to the algebraic signs +, −, or 0 of the coefficient of the *i* term in equation (5a).

Case 1: The disk turns slowly ($\omega < 2R/r^2\mu_o n$). Equation (5a) can be written as

$$\frac{di}{dt} = -\frac{i}{T^*} \quad (8)$$

where

$$T^* = \frac{L}{|{}^1\!/\!_2 r^2 \omega \mu_o n - R|}. \quad (9)$$

The solution of equation (8) is

$$i(t) = i_o e^{-t/T^*}. \quad (10)$$

This decreasing exponential is shown as curve 1 in Figure 3. The current dies out, because the disk turns too slowly to generate enough current to maintain the magnetic field.

The power expended by the external agent forcing the disk to turn at a constant angular velocity is

$$P = \tau\omega. \quad (11)$$

When equations (6) and (11) are used in the above formula, the power expended by the external agent is

$$P = {}^1\!/\!_2 \mu_o n r^2 i_o^2 e^{-t/2T^*}. \quad (12)$$

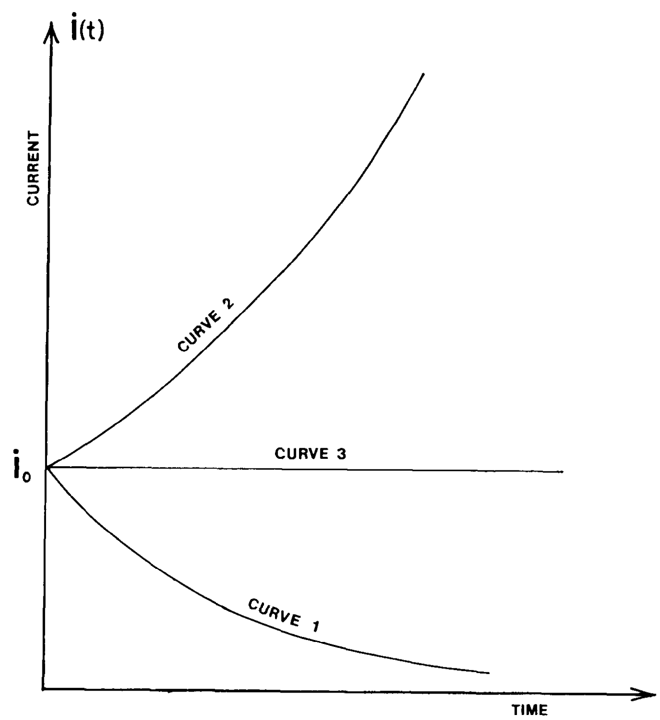

Figure 3. Behavior, as a function of time, of the current in the disc of a Faraday-disc dynamo, turning at a constant angular velocity.

Consequently, the disk becomes easier and easier to turn as the current dies out.

Case 2: The disk turns rapidly ($\omega > 2R/r^2\mu_o n$). Equation (5a) can be written as

$$\frac{di}{dt} = +\frac{i}{T^*}. \tag{13}$$

The solution of equation (13) is

$$i(t) = i_o e^{+t/T^*}. \tag{14}$$

This increasing exponential is shown as curve 2 in Figure 3. The current continually grows, because the disk turns so rapidly that more than steady state current is produced.

The power expended by the external agent is found to be

$$P = \tfrac{1}{2}\mu_o n r^2 i_o^2 e^{+t/2t^*}. \tag{15}$$

This runaway solution corresponds to a physically unrealistic situation. To force the disk to maintain its high constant angular velocity, the external agent must supply exponentially increasing amounts of power. Eventually, a practical limit would be reached. Thus, case 2 must be ruled out as a physically possible solution for the long-term behavior of the dynamo, even though it is mathematically correct. Of course, for short periods of time, the external agent could supply the exponentially increasing power required.

Case 3: The disk rotates with $\omega = 2R/r^2\mu_o n$. Equations (5a), (7), and (11) yield

$$i = i_o = \text{constant},$$

$$P = \tfrac{1}{2}\mu_o n r^2 i_o^2 = \text{constant}.$$

The external agent supplies constant power. The current generated in the disk is just right to produce exactly the magnetic field needed to generate the current. This solution is represented as curve 3 in Figure 3.

An estimate of the numerical value for the angular velocity separating these three cases can be made using $n = 1000$ turns/m, $r = 0.1$ m, $L = 10$ henry, $\mu_o = 1.26 \times 10^{-6}$ henry/m, and $R = 10$ ohm. For these values, $\omega = 2R/r^2\mu_o n = 1.6 \times 10^6$ rad/sec which corresponds to one quarter of a million revolutions per second. For a laboratory-sized dynamo, only case 1 would be reasonable. For a dynamo the size of the earth's core, $\omega = 2R/r^2\mu_o n$ might correspond to a number on the order of one revolution per day, so that all three cases might correspond to reasonable values for ω for a restricted period of time. Even so, for the earth-sized dynamo, case 2 must still be rejected as a physically unrealistic solution for the long-term behavior for the dynamo.

In all three cases, an unanswered question is the origin of the original current and magnetic field. In the laboratory, this original magnetic field and current could be produced by quickly withdrawing a bar magnet from the coil of wire. However, the origin of the earth's magnetic field cannot be explained by such a mechanism.

It might be added that laboratory models may have a permanent magnet, not an electromagnet. Obviously, however, that would not help the case of the earth's magnetism. For it is a magnetic field which has to be explained. Any field introduced to explain it would have in turn to be explained, and so on to infinity; one would fall into an infinite regress.

IV. Free-Running Dynamo

If there is no external agent to supply power to the dynamo, it must power itself. (Or, rather, dissipate the energy originally imparted to it, then stop.) The flow of current is governed by equation (5b), the magnetic field is given in terms of that current by equation (3), and the motion of the disk is governed by equation (7). When the mechanical equation (7) and the electromagnetic equation (5b) are combined, the result is

$$\left(\frac{r^2\mu_o n}{L}\omega - \frac{2R}{L}\right)\frac{d\omega}{dt} - \frac{d^2\omega}{dt^2} = 0. \tag{16}$$

The substitutions $T = L/R =$ a time constant; $u = \omega T$; $a = r^2\mu_o n/L =$ a pure number; and $x = t/T$, reduce the coupled equation to the dimensionless form

$$(au - 2)\frac{du}{dx} - \frac{d^2u}{dx^2} = 0. \tag{17}$$

Letting $y = u - 2/a$ the coupled equation becomes

$$\frac{d}{dx}\left(\tfrac{1}{2}y^2 - \frac{1}{a}\frac{dy}{dx}\right) = 0, \tag{18}$$

or

$$\tfrac{1}{2}y^2 - \frac{1}{a}\frac{dy}{dx} = c', \tag{19}$$

where c' is a constant of integration. The final integral of equation (19) depends on the sign of the integration constant, c'. Fortunately, the sign of c' can be determined from mechanical considerations. It was shown above that dy/dx is proportional to $d\omega/dt$; in fact $dy/dx = T^2 d\omega/dt$.

The torque equation (7) requires

$$\frac{d\omega}{dt} = -\frac{\mu_o n r^2 i^2}{2I} < 0. \tag{20}$$

Thus.

$$\frac{dy}{dx} = -\frac{T^2\mu_o n r^2 i^2}{2I} < 0. \tag{21}$$

Therefore, c' in equation (19) is equal to the sum of two non-negative terms, $\tfrac{1}{2}y^2$ and $-(1/a)(dy/dx)$, so c' must be a non-negative constant. Therefore, equation (19) will be written as

$$\tfrac{1}{2}y^2 - \frac{1}{a}\frac{dy}{dx} = \tfrac{1}{2}c^2. \tag{22}$$

Physically, the sign of c is determined by the condition that the disk always experience a resistive electromagnetic force. For this reason, even $c = 0$ is excluded in equation (22). Equation (22) is solved as follows:

$$\int_{y_o}^{y} \frac{dy'}{(y'^2 - c^2)} = \frac{a}{2}\int_o^x dx',$$

$$-\frac{1}{c}\left\{\tanh^{-1}\left(\frac{y}{c}\right) - \tanh^{-1}\left(\frac{y_o}{c}\right)\right\} = \frac{ax}{2},$$

$$y = c\tanh\left[-\frac{acx}{2} + \tanh^{-1}\left(\frac{y_o}{c}\right)\right]. \tag{22'}$$

When the definitions around equation (17) are used, the above solution in terms of the original quantities is

$$\omega = \frac{c}{T} \tanh \{ -\frac{act}{2T} + \tanh^{-1} (\frac{\omega_o T}{c} - \frac{2}{ac}) \} + \frac{2}{aT}. \quad (23)$$

When this solution for ω is substituted back into equation (7), the current i is found to be

$$i = \pm \frac{c}{T} (\frac{I}{L})^{1/2} \operatorname{sech} \{ -\frac{act}{2T} + \tanh^{-1} (\frac{\omega_o T}{c} - \frac{2}{ac}) \}. \quad (24)$$

The ± sign ambiguity describes the fact that the current could flow in either direction, and it is a mathematical consequence of equation (7) determining i^2, not i directly. Equation (24) is used to determine the constant of integration c in terms of the initial current i_o as follows:

$$i_o = \pm \frac{c}{T} (\frac{I}{L})^{1/2} \operatorname{sech} \{ 0 + \tanh^{-1} (\frac{\omega_o T}{c} - \frac{2}{ac}) \}. \quad (24')$$

Use of the identity $\tanh^2 \theta = 1 - \operatorname{sech}^2 \theta$, with $\theta = \tanh^{-1} (\omega_o T/c - 2/ac)$, yields

$$c = [(\omega_o T - \frac{2}{a})^2 + \frac{LT^2 i_o^2}{I}]^{1/2}. \quad (25)$$

It is interesting to note that the current will undoubtedly die out, since equation (24) predicts $i \to 0$ as $t \to +\infty$, but the disk will probably continue to rotate. The final angular velocity ω_f as given by equation (23) in the $t \to +\infty$ limit is

$$\omega_f = \frac{(2/a - c)}{T}. \quad (26)$$

Thus, the disk will (1) slow down, but continue to rotate in the same direction if $2/a > c$, (2) slow to an eventual stop if $2/a = c$, or (3) reverse direction once and only once approaching a constant reverse rotational velocity if $2/a < c$. Which of these three fates awaits the disk is determined by the properties of the material, the geometry of the disk and coil, and by the initial values of angular velocity and current (or magnetic field). However, the current i as given by equation (24) along with the magnetic field (which is proportional to i by equation (3)), *never* reverses direction. The ± sign ambiguity in equation (24) does not indicate or allow a current or a field reversal. Rather this sign ambiguity indicates that the initial current and magnetic field could be in either direction, regardless of the direction of rotation of the disk. However, once the direction of the current and magnetic field are determined from the initial conditions, the sign ambiguity is removed, and there is neither current nor field reversal.

It is instructive, though, to examine the unphysical solution obtained with a negative integration constant c' in equation (21), say $c' = -\frac{1}{2}k^2$ where k is a real constant. In this case, equation (21) becomes $y^1/2 - (1/a)(dy/dx) = -k^2/2$. The solution is obtained as follows:

$$\int_{y_o}^{y} \frac{dy'}{(k^2 + y'^2)} = \frac{a}{2} \int_o^x dx'$$

$$\frac{1}{k} \{ \tan^{-1} (\frac{y}{k}) - \tan^{-1} (\frac{y_o}{k}) \} = \frac{ax}{2}$$

$$y = k \tan \{ \frac{kax}{2} + \tan^{-1} (\frac{y_o}{k}) \}. \quad (27)$$

In terms of the definitions around equations (16) and (17) this solution is

$$\omega = \frac{k}{T} \tan \{ \frac{kat}{2T} + \tan^{-1} (\frac{\omega_o T}{k} - \frac{2}{ka}) \} + \frac{2}{aT}. \quad (28)$$

When this solution for ω is substituted back into equation (7), the current i is found to be

$$i = (-1)^{1/2} (\frac{I}{L})^{1/2} \{ \frac{k}{T} \sec (\frac{kat}{2T}) + \tan^{-1} (\frac{\omega_o T}{k} - \frac{2}{ka}) \}. \quad (29)$$

The magnetic field B, from equation (3), is proportional to this current i. In this case, if the result were meaningful, both the magnetic field and the current would reverse each time the argument of the trig function increases by π, or $\Delta t = 2\pi T/ka$. However, the physically fictitious nature of this mathematically correct solution is evident. The current is proportional to $(-1)^{1/2}$ and so it is purely imaginary. Thus, although there are mathematically acceptable reversing-field solutions, there are no physically acceptable ones.

V. Conclusions

The Faraday disk dynamo provides a solvable, simple model of the dynamo theory advanced to explain the source of the earth's magnetic field. The coupled electromagnetic and mechanical effects are clearly illustrated by the Faraday disk dynamo. To this extent, the Faraday disk dynamo shows the possibility of geomagnetism from a rotating dynamo effect in the earth's core. It does not prove that such a dynamo in fact exists. However, the field reversals hoped for by the paleomagnetists are denied by the disk dynamo. Although there are field-reversing solutions, such as equation (29), that are mathematically correct, these field-reversing solutions are purely imaginary, hence unphysical or physically impossible. The only mathematically real, hence physically possible, solutions are given by equations (23) and (24). These solutions allow a reversal of the disk's motion once, but never a reversal of the magnetic field direction. Thus, some explanation other than geomagnetic field reversals should be sought to explain reversed magnetized rocks. A return to the recent creation geomagnetic model of Lamb as revised by Barnes would offer a better explanation of geomagnetism. These models at least explain the observed decrease in the strength of the earth's magnetic field over the past century and a half. These are known magnetic field strengths measured at known times, not inferred from reverse magnetized rocks whose reverse magnetism could be due to other effects.

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