SOME SIMULATIONS OF THE POSSIBLE ROLE OF CAVITATION IN CATASTROPHIC FLOODS*

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Abstract

The process of cavitation in water has been involved in the damage of many types of man-made structures. Flow speeds greater than 30 m/s appear necessary for cavitation damage. Major damage can occur with flow depths of only a few meters but it decreases with flow depth, channel roughness and air bubble content of the water.

A *computer model predicting damage potential, calibrated qualitatively with actual damages to dam spillways, is used to indicate the locations and relative intensity of damage for several spillway profiles. While damage is more likely associated with steeply sloping channels, because of the high flow velocities achieved in them, damage can occur in nearly horizontal surfaces if there is some other mechanism for achieving the necessary flow speeds. In a hypothetical spillage of water over the rim of the Grand Canyon, there are numerous locations at which cavitation destruction of the rock would be as great or greater than the worst damage ever seen in actual dam spillway.* A *flow of water of only four meters (m) initial depth, approaching a rapid elevation drop of less than 100 m at an initial speed of only 10 m/s can be expected to produce major cavitation damage for a variety of natural land profiles. The process of damage by cavitation appears to be a likely mechanism for rapid removal of rock in channels experiencing catastrophic flows of high speed shallow water with little air bubble content.*

Introduction

Cavitation is the creation of gaseous phase bubbles in a liquid as a result of a decrease in pressure. While the creation of the bubbles themselves is relatively harmless, it is the *collapse* of bubbles that can cause structural damage to surfaces that are in contact with the liquid. It will cause powerful shockwaves and possibly minute jets of water that impact on the solid surfaces. Though the collapse of the bubbles is actually the opposite of the creation of vapor cavities, the term cavitation tends to be used to refer to the entire process. The U.S. Bureau of Reclamation conducted extensive studies to be able to understand the conditions under which cavitation damage might occur, to predict the location and severity of damage, and to design corrections to prevent future damage to the water conveyance structures. Falvey (1990) suggests that water heads in

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See** **a technical presentation on the same subject by Holroyd,** ***CRSQ **27:23-32.**

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excess of about 45 meters and flows in excess of 30 m/s are suspect for the potential for producing damage to structures by cavitation. They found that it was possible to control the curvature of spillways in the design process so as to minimize the possibility of cavitation damage. It was also found that the injection of air bubbles into the water flow stopped the damage under normal operating conditions. Holroyd (1990) summarized the findings of the monograph along with descriptions of other phenomena related to cavitation. Along with the text, giving all relevant equations and some calibrations using actual structures, the monograph (Falvey, 1990) includes a set of 5.25 inch floppy disks for use on an IBM compatible microcomputer. The original pro ram code is provialed in FORTRAN. One piece of software receives as input a nominated initial flow condition and structural profile. The output is a table of flow conditions throughout the structure, including parameters relating to cavitation. Some of the outputs can be graphs of a few parameters. Additional programs provide guidance for profiles having a constant cavitation index, for design of aerator slots for injecting air bubbles into the flow, and for estimating the damage index from a record of historical flow conditions.

One of the computer programs supplied with the monograph was run, first on the Glen Canyon Dam tunnel profile, where damages were measured, and then on several other profiles, man-made, artificial, and natural. These computer model runs give guidance on the possibilities of cavitation damage to water channel surfaces during flow conditions greatly exceeding those experienced during the 1983 floods.

Cavitation Parameters

There are three numbers among the equations given by Falvey (1990) that are used to describe several aspects of the cavitation process: the cavitation index, the cavitation potential, and the damage index. It is challenging for those who do not use the terms regularly to distinguish between them. The definition equations are repeated in Holroyd (1990).

The cavitation index has as its numerator the difference between the ambient water pressure and the vapor pressure of water at its particular temperature. The denominator is the ambient kinetic energy of the water and therefore is proportional to the square of the water velocity. A small cavitation index indicates that it is relatively easy for some flow disturbance to vaporize water.

The damage potential addresses the question, given that cavitation is likely to occur at a location, of how strong the damaging forces will be. Falvey (1990) points out that the damage potential is inversely proportional to the cavitation index and appears to be proportional to the sixth power of the water speed. The definition equation compares the cavitation index and flow velocity to reference values for the threshold of cavitation. The damage potential was crudely calibrated in the Falvey monograph by comparing actual damage at several dams with the theoretical damage potential numbers. He gives values of damage potential for "incipient;" "major," and "catastrophic' damage as 500, 1,000 and 2,000, respectively. The damage at Glen Canyon Dam in 1983, in which the water forces consumed reinforced concrete one meter thick and excavated sandstone bedrock to a depth of nine more meters, was considered to be "catastrophic." It is the behavior of the damage potential that will be examined later in this report.

The damage index is an estimate of the cumulative damage produced by cavitation. It is defined as the damage potential times the logarithm of elapsed time. It is likewise crudely calibrated at 5,000, 10,000, and 20,000 for incipient, major, and catastrophic cavitation damage, respectively. The damage index recognizes that cavitation is self-limiting. As the solid surface is destroyed by the forces of cavitation, that surface tends to recede from the location of the collapsing bubbles and becomes less susceptible to further damage.

Computer Simulations

The Falvey software presents theoretical cavitational characteristics for a variety of flat and curved profiles of water channels, surface rugosities, flow depths and speeds, and several sizes and shapes of flow disrupters. That part of the software for eliminating cavitation damage by better design of curves and design of aerator slots is useful for construction purposes only; it is of limited usefulness for investigating the response of existing natural structures to flows capable of producing cavitation damage. The other programs describe and graph the flow and cavitation conditions for any nominated profile and initial flow conditions. The software has been run for over 20 Reclamation-designed dams. Experience with structures in other parts of the world has also been considered, including some in which no cavitation damage has ever been observed. The software is currently able to predict locations of cavitation damage with high reliability. Though more research and calibration is probably in order, the present computer model can be considered to be approximately calibrated for real flow conditions. Its outputs are in good agreement with observations.

There are limitations to the software, however. It is only a one-dimensional model with presently up to 40 profile data points allowed as input. The array dimensions could be increased and the FORTRAN program recompiled. But to change it to a two- or three-dimensional model would require major surgery and the result may require a computer much bigger than a microcomputer. The one-dimensionality presently means that the program only deals with conditions in the direction of the flow. It does not address lateral flows or longitudinal vortices. Furthermore, the computational results of most computer models must be considered suspect when the models are pushed to conditions outside the ranges for which they were calibrated. This model gives complaints in the output when it is subjected to conditions likely to produce erroneous results. Therefore the outputs of this model should only be considered to be qualitative at the upper and lower extremities of the ranges considered.

The input table requires a consecutive listing of surface elevations versus downstream distances (stations). A variety of channel cross sections are allowed, and they must be described at each station. The vertical radius of curvature at each station is also required. For man made spillway structures, such radii can be gleaned from design speciifications. But for those seeking to input natural channel profiles as determined from topographic maps, the local radius of curvature must be calculated separately. A special computer program was written by Holroyd and included with the software package for that purpose. Its main equations are given in the Appendix.

The sample data set that comes with the software describes the profiles of eight structures. The computer operator can easily change initial flow rate (volume/see), initial depth at the top of the structure, and surface roughness. The entire profile can be changed to anything else. The program will tell where it needs intermediate data points, up to a total of 40 in the profile.

However an interested person may find it easier to relate to flow speed rather than to flow rate. Therefore most of the following simulations express downstream cavitation damage potential as a function of location and initial depth for certain initial flow speeds rather than flow rates. For a given channel cross sectional area, any two of depth, speed, and flow rate determine the third quantity. For constant flow rate, the depth and speed are generally inversely proportional, with some adjustments resulting from channel cross sectional shape. The initial flow rates of these simulations were those determined by the nominated initial depths and speeds.

The “catastrophic” flows through the Glen Canyon Dam drainage tubes were small compared to flows that those interested in catastrophic geological processes would like to consider. Therefore the software was used to explore greater magnitudes of flows. The Glen Canyon left tube model was subjected to flows from so low that the software complained of analysis difficulties up to the limit of the design capacity of the tubes. Then the left tube profile was retained but the cross section was changed from circular to rectangular of slightly greater width and unlimited top. Again the flows were varied from near the lowest to near the highest that the software would accept. Then, to minimize the effects of the side walls, they were moved out to a separation of 1000 ft (305 m). Such variations on the Glen Canyon profile showed that the model could be extrapolated into conditions unlikely to be experienced today. The integrity of the results during such modifications gave confidence that the next modifications to artificial and natural profiles would give reasonable results. In this way there is an orderly progression from known cavitation conditions to those far beyond present day experiences.

Glen Canyon Left Spillway Simulations

The initial flow and flow depth at the top of the left spillway of the Glen Canyon Dam were varied over a wide range, subject to the ability of the computer program to describe the resulting flows farther down the spillway. Very shallow flows ran into boundary layer problems and then could not climb up the flip bucket at the end of the tunnels. (The flip bucket is a curved surface at the end of a spillway that launches the water into the air on a trajectory that makes it fall into a pool of water and dissipate its energy there.) Very large flows sometimes choked the tunnel. The program output provides numerous parameters related to cavitation. Only the speed profiles and the damage potential are presented here.

The program was stepped through an orderly series of initial depths with the initial flow rates for each computer run carefully adjusted to produce initial speeds of 5, 10, and 20 m/s at the top of the spillway. Subsequent speeds are presented in Figure 1 with respect to distance downstream of a standard reference point. The water rapidly accelerates as it falls down the spillway. After passing the 700 m mark (see Figure 5, Holroyd, 1990, for the tunnel and damage profiles) the tube bends to the horizontal (starting about 800 m). The sudden transition between the centrifugal pressures of the curved profile and the nearly static pressures of the horizontal flow produces a forward acceleration and suddenly higher velocities there. It is also the location of the deepest damage. Figure 1 also shows that the shallowest flows have their speeds reduced by the friction of the boundary layer. Flows greater than 4 m initial depth became choked at the slow 5 m/s initial speed. Throughout the profiles the subsequent depth of the flows tended to be inversely proportional to the flow speed at each location. The combination of fastest speeds and shallowest depths near the 800 m location is what set up the damaging conditions there.

The damage potential for the same initial flow speeds and depths are given in Figure 2. For this figure and all of those to follow, the damage potential is for a surface perturbation consisting of a 10 mm

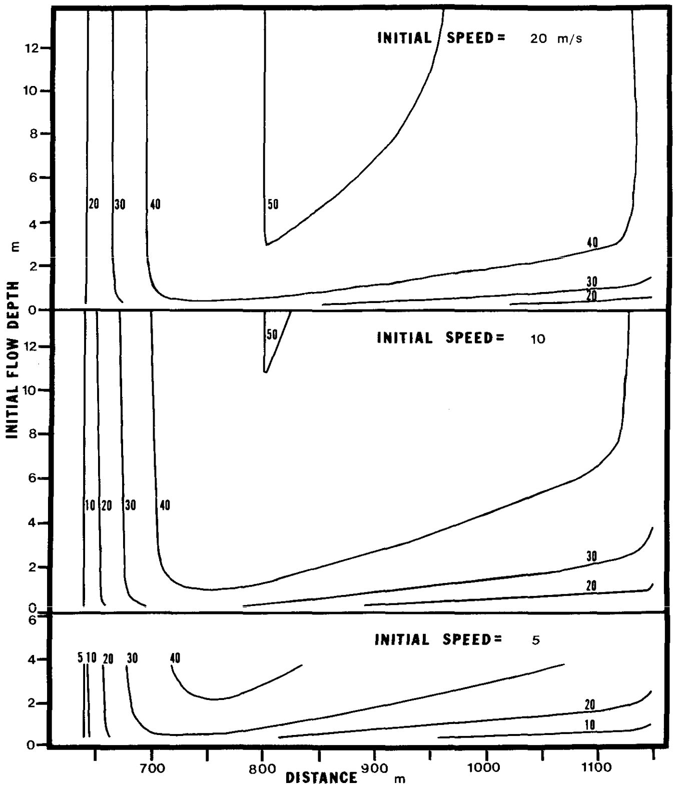

Figure 1. The flow speeds, all in m/s, along the left spillway tunnel for a variety of initial water depths and three initial speeds at the top of the spillway.

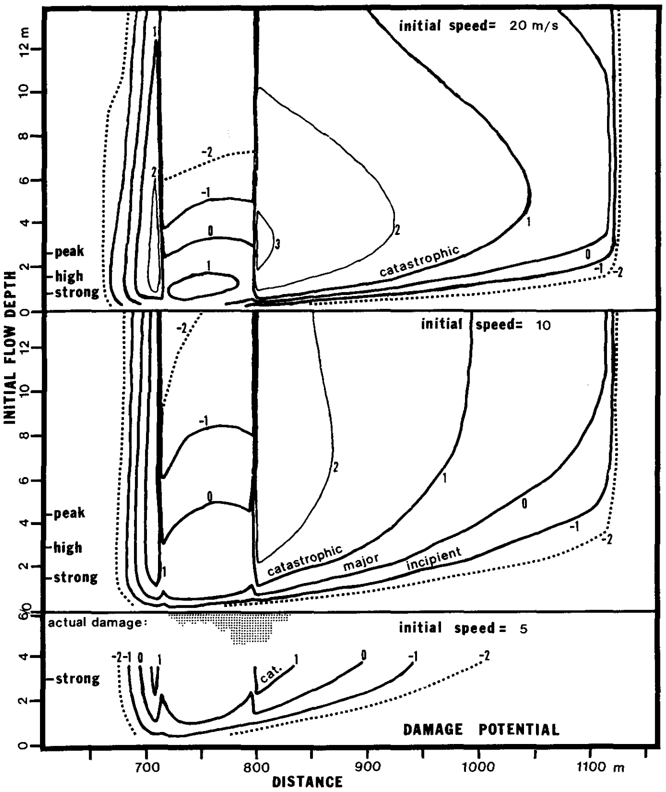

Figure 2. The damage potential for the Glen Canyon Dam left spillway tunnel as a function of initial depth. The low velocities are those given in Figure 1. The damage potential is labeled in terms of powers of two times "major" cavitation damage potential.

(1/2 inch) circular arc. At about this size the damage potential is approximately proportional to the size of the bump. (It is amazing that a bump as small as 10 mm can initiate a chain of craters leading up to hole 10 m deep.) The program output also gives the values at other bump sizes and shapes. The rugosity value was that for very smooth concrete. For this and subsequent figures the damage potential contours are given number labels, n, on a logarithmic scale, where the damage potential is 1000 x 2^n. With this coding a -1 is for incipient cavitation, 0 for major and 1 catastrophic, according to the Falvey definitions. A number of **11,** to be seen in a later figure, would then indicate a damage potential 1024 times larger than the "catastrophic" of the 1983 Glen Canyon Dam flows.

At the top of the graph for the 5 m/s initial speed the actual cavitation damage from Holroyd (1990) Figure 5, in terms of relative depth, is shown by the shading. At the left of all three parts of the Figure are shown indicators for the initial flow depths corresponding to 300 (strong), 600 (high) and 900 (peak) m^3/s flow rates, corresponding to Holroyd (1990) Figure 2. The Falvey monograph does not give the initial flow speeds for the historic flow rates. The failure of the computer program to model the higher flow rates at the 5 m/s speed suggests that the actual speeds might have been between 10 and 20 m/s. For later comparisons the 10 m/s initial speed was selected as a reasonable value.

Cavitation damage is not the same as damage potential. Though there was only one major peak of damage near the 800 m location, the curves of Figure 2 indicate two peaks for damage potential, at the start and end of the circular transition from near vertical flow to near horizontal flow. Damage potential is greatly reduced in the circular bend because the centrifugal forces produce larger pressures, making cavitation less likely. Of all of the dam profiles tested by the Bureau of Reclamation with this program, only the upper peak in damage potential in the Glen Canyon Dam spillway failed to be actualized. Furthermore, while there is less damage potential in the circular bend than in the transitions on either end, damage still occurred there, as illustrated in Holroyd (1990), Figure 11. A change from smooth concrete to a rough concrete surface (not shown) decreased the calculated damage potential.

Figure 2 also illustrates some of the theoretical behaviors discussed in Falvey (1990) and reviewed in Holroyd (1990). The shallow initial flow depths produce even shallower depths downstream. Friction limits the speeds of the shallow depths of water and therefore causes the steep gradient of damage potential with depth for initial depths of less than 2 m. The 20 m/s diagram shows that increasing initial depth actually *decreases* the damage potential because increasing depth increases static pressure and makes it more difficult for dynamic pressure reductions to reach values at which-water will vaporize. The effect of increasing initial depth on decreasing cavitation damage potential is especially strong in the circular curvature section between 700 and 800 m, where centrifugal forces add to the static pressure. This is a strong reminder that great depths of water will not cavitate. An ocean of water traveling across land at high speeds may be quite destructive, but it will not be by cavitation.

The Wide Spillway Simulations

With the behavior of the circular cross section profile of the Glen Canyon Dam Left Spillway established, the cross section was first modified to a rectangular profile with flat bottom and vertical sides of unlimited extent. The vertical profile was retained and the width was initially increased to 50 ft. The changes in the model outputs were relatively minor. The width was then increased to 100 ft. (305 m) to simulate a widespread flood going over the same vertical profile. The flip bucket was removed and a gently sloping approach to the top of the profile was added. In this way it was sought to minimize the model's sensitivity to the side walls of the channel. No limit was placed on the depth of the water in this wide channel, but the computer model itself indicated upper and lower limits to its ability to simulate the flows. Figure 3 shows the distributions of damage potential, coded as before, for only the 10 m/s initial flow speed, as a function of initial depth and downstream distance. The results are similar to Figure 2 but with less of a decrease in damage potential in the 700 to 800 m zone. Flow depths sufficient to reverse the damage potential with depth were not included in this simulation.

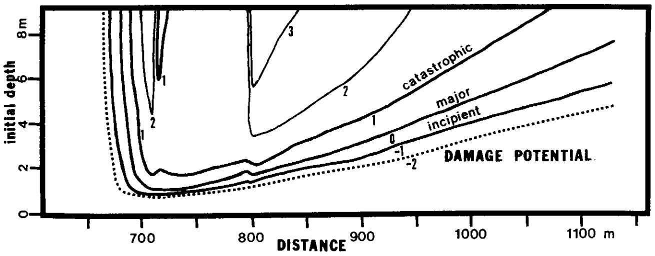

Figure 3. The distribution of the damage potential with initial depth and downstream distance for a 10 m/s initial flow speed. The damage potential is coded as in Figure 2. The Glen Canyon Dam left spillway profile was modified to eliminate the flip bucket at the end, add a gently sloping approach to the top of the spillway, and widened to a 1000 foot rectangular cross section with vertical walls.

The Semi-horizontal Simulations

Having established that wide flows can be examined and that they give similar results to the circular cross section investigations, the possibilities of cavitation over nearly flat terrain were examined. Input parameters were chaned to metric, with a width of 1 km. An artificial profile was designed which consisted of 5 km of nearly flat terrain having a constant slope of only 0.004. Then, to drain the water away from the semi-flat region, the terrain was given a parabolic profile matching the free fall arch of an object traveling horizontally at 500 m/s. Initial flow speeds of up to 100 m/s were used in the simulations and the conditions in the parabolic section were ignored. The water was aways decreasing its speed over the semi-flat portion and therefore increasing its depth to maintain a constant flow.

Numerous computer runs were made in order to map out the damage potential presented in Figure 4. In the Figure it is the *actual* water depth that is the ordinate and the *actual* flow speed for the abscissa rather than the initial conditions as in the earlier Figures. The logarithmic coding of the damage potential is again used. The solid lines give the locations of the calibrated "incipient, major, and catastrophic" conditions and the dotted lines the extrapolated values. It is seen that significant damage potential begins with shallow flows of about 30 m/s, according to this simulation. It increases rapidly with flow speed, remembering that the contour lines differ by a *factor* of two. It is also seen that the damage potential decreases with increasing flow depth. This is because the increased ambient pressure caused by greater depths remakes it harder for dynamic pressure fluctuations to reduce to the vapor pressure of the water.

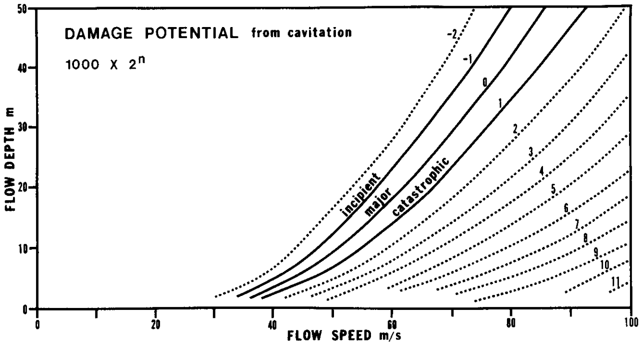

Figure 4. The damage potential for nearly flat terrain for actual, rather than initial, flow depths and speeds.

In creating this Figure, no consideration was made as to how such velocities might be achieved. Indeed, the rapid reduction of speed with downstream distance that appeared in these simulations indicates that viscosity and friction will prevent high speeds from being sustained at a slope of 0.004. Yet Figure 4 indicates that if there is some cause for water to exceed a speed in excess of about 30 m/s, then cavitation damage potential exists for even semi-flat terrain. The nearly flat terrain of this simulation was especially designed to produce a simplified diagram in which the speed threshold of cavitation damage potential and the normally inverse relationship of flow depth to damage potential were readily evident.

The Papago Creek (Grand Canyon) Simulations

There are an infinite variety of profiles that can be investigated with the model. For this report a natural channel was chosen to see if there were any conditions under which cavitation would occur if various flows of water were allowed to follow such a profile. In order to get depth and variety, a steep and generally straight side channel of the Grand Canyon of Arizona was chosen. The profile was along a line parallel to Papago Creek (about 36° 2'N, 111° 54' W), from the highway towards Solomon Temple, at an azimuth of 333°. The horizontal distance was the distance of the contours from the highway, not the integrated distance along the twisting channel. The resulting profile and the geologic strata, derived from a geologic map (Museum of Northern Arizona 1986), of the cross section are shown in the upper part of Figure 5 and Table 1. The elevations were taken from a 1:62500 scale topographic map of 80 feet contour interval. Though nearly 60 contours were available, only 40 could be used in the model. So 160 feet contours were used initially and then the intermediate contours were used in sections (generally flat) recommended by the software.

The contours of damage potential in Figure 5 have the same coded labels as the previous Figures. Though the contours are not always resolvable in this reproduction, what is important is their density and location. The middle drawing has a rugosity and initial conditions comparable to those in Figures 2 (center) and 3. Yet the damage potential generally equals or greatly exceeds that for the Glen Canyon Dam spillway profiles. The curve labeled 8 in Figures 4 and 5 indicates a damage potential exceeding 100 times the "catastrophic" conditions observed in the 1983 floods. The lower part of Figure 5 is probably more realistic for a natural channel roughness. Yet it also indicates the possibility of equal or greater damage from cavitation than observed in 1983 at the Glen Canyon Dam. As in Figure 3, the water depths were not increased to the level beyond which the cavitation potential would decrease. The cliff-like portions of the refile made the software issue complaints for depths greater than those illustrated.

Comparing the upper and lower parts of Figure 5 gives an indication of which parts of the profile are likely to cause cavitation damage. The highest damage

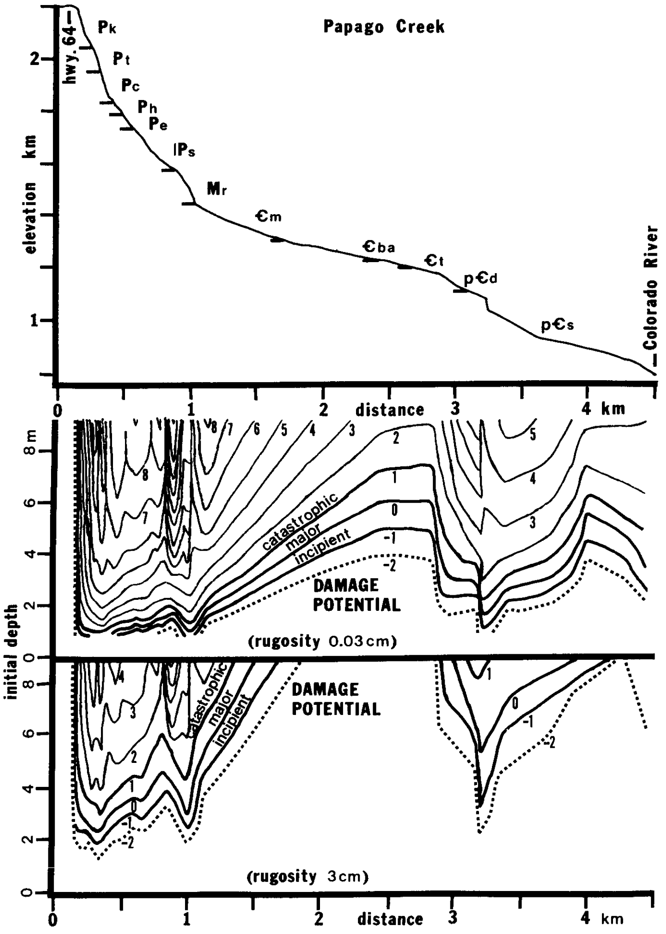

Figure 5. (Top) The vertical profile of Papago Creek and the geological strata exposed, using the symbols in Table I. The distribution of cavitation damage potential as a function of initial depth and downstream distance or a 1000 ft wide Papago Creek profile for 10 m/s initial flows speed. The damage potential is code as in Figures 2-4 for a smooth surface (middle) and rough surface (bottom).

potential occurs where water would encounter a negative radius of curvature for the surface. This condition reduces the ambient pressure (weight of the water above a point) much like a vehicle traveling over the same profile experiences a tendency towards weightlessness. But such locations can also inject air into the water stream if they are cliff-like, such as the Redwall limestone near the one km location.

Other locations for enhanced damage potential are where the water has a great speed from a recently rapid drop in altitude. On the other hand, the reduced damage potential from 2 to 3 km results from reduced speeds and increased flow depths caused by the more level terrain. In general, the damage potential is greatest where there are steep drops in the stream profile. This suggests that the heads of canyons can experience rapid removal of rock as a result of cavitation processes if such large flows of water spill into them without much ingestion of air.

The choice of Papago Creek was made to find out what a variety of rock strata would do, as represented by present profiles. Hard rocks will have semi-horizontal top surfaces and cliff-like edges. Softer shales will have intermediate slopes. If a large flow of water was to pass over the cliff edge in such an environment without ingesting much air, then cavitation-initiated rock destruction is likely to occur. The choice of Papago Creek was for convenience. It does not indicate any suggestion that a large flow actually occurred in that location. There is presently no way such a flow of water could arrive at the cliff edge and the size of the headwaters is trivial. However, the harnesses of the rocks during the carving of the Grand Canyon might have been similar to those observed today and reflected in the present erosion profile. A catastrophic flow of water, such as might result during the capture of the Colorado River through the Kaibab uplift, might encounter similar profiles. This computer simulation shows that there are indeed locations for cavitation processes to greatly accelerate the removal of rock.

Table I. The sequence of rock formations along the Papago Creek profile shown in Figure 5.

Symbol	Formation
P_k	Kaibab limestone
P_t	Toroweap
P_c	Coconino sandstone
P_h	Hermit shale
P_e	Esplanade sandstone
IP_s	Supai
M_r	Redwall limestone
Є$_{m}$	Mauv limestone
Є$_{ba}$	Bright Angel shale
Є$_{t}$	Tapeats sandstone
pЄ$_{d}$	Dox sandstone
pЄ$_{s}$	Shinumo Quartzite

Discussion and Conclusions

The previous article (Holroyd, 1990) summarized the physics of the process of cavitation. As considered in this paper, cavitation is the creation of water vapor bubbles within liquid water by the reduction of pressure to the vapor pressure of water at the temperature of that water. The term cavitation has been erroneously extended to include the damaging processes associated by the collapse of those bubbles.

The process of cavitation damage relating to water conveyance structures was explored with the help of a monograph on cavitation and the accompanying software packages. The software, qualitatively calibrated by assessments of historical damage to existing spillways, was used to map flow and cavitation conditions for the Glen Canyon Dam left spillway tunnel. After documenting the software behavior for known damage, the programs were used for other simulations. Changing from circular cross section to rectangular of 1000 feet width showed similar cavitation behavior. The simulation of wide, nearly flat, terrain illustrated the effects of water speed and depth on he cavitation process. It indicated that cavitation damage should be suspected for flow speeds greater than 30 m/s in shallow water. The simulated spill of water over the rim of the Grand Canyon indicated a potential for greater cavitation damage to the rock than the greatest damage observed in the 1983 floods.

The purpose of these initial simulations was to examine the cavitation process for a few profiles during conditions of water flow much greater than commonly experienced in recent floods. Though the monograph labeled the 1983 Glen Canyon Dam spillway damage as “catastrophic,” the software was pushed to damage potentials over 100 and 1000 times as great, for conditions that have never been measured. The software cannot be calibrated for those conditions, but it still gives guidance for such extremes. As expected, flows in excess of those observed in 1983 can be expected to produce damage much more severe than that which created automobile-sized boulders out of sandstone bedrock during a several-week period. This suggests that great flows of water might have the potential for carving canyons even in hard rocks during several weeks rather than the slower thousands-of-years rates observed with normal erosion processes. Yet the simulations also showed that eventually cavitation damage potential decreases with increasing water depth.

This has been only an initial exploration of cavitation with software simulations. The variety of possible input conditions is practically infinite. There is opportunity for further research into numerous phenomena related to the process of cavitation. The software can be run on many natural profiles for water channels to explore the range of profiles and water flows that might lead to cavitation conditions. Of particular interest might be scenarios of catastrophic drainage of post-glacial lakes. There are several canyons which, if plugged, would support vast lakes upstream of them. A breaching of the dam, a natural ridge holding back the lake waters, could involve water speeds of sufficient to initiate cavitation damage. Thousands or millions of years are not necessarily needed for the carving of some valleys and canyons if the process of cavitation becomes involved.

Acknowledgements

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Museum of Northern Arizona 1986. Geologic map of the eastern part of the Grand Canyon National Park, Arizona. Grand Canyon Natural History Association. Grand Canyon, AZ.

Appendix. Radius of Curvature

The program calculates the vertical radius of curvature for three pairs of consecutive stations (x_1, x_2, x_3) and elevations (y_1, y_2, y_3). For a subsequent calculation the first point is dropped when the next pair is entered. The radius of curvature is assigned to the middle point, like what is done for the calculation of a running mean of three points. The three points form a triangle whose sides a, b, and c can be determined by:

$$\begin{aligned} a^2 &= (x_1-x_2)^2 + (y_1-y_2)^2, \\ b^2 &= (x_2-x_3)^2 + (y_2-y_3)^2, \\ c^2 &= (x_3-x_1)^2 + (y_3-y_1)^2. \end{aligned} \tag{1}$$

Following a formula in Hodgman (1938) for the radius of a circle circumscribed around a triangle, the half perimeter, s, and a subsequent product, t, are

$$s = (a + b + c)/2, \quad t = s(s-a)(s-b)(s-c). \tag{2}$$

To protect against illegal calculations, if c >= (a + b) or t =< 0, then the radius of curvature, R, is set to zero. Otherwise

$$R = abc/4(t)^{½} \tag{3}$$

The sign of the curvature must then be determined, where concave upwards (center of curvature above the flow surface) is considered positive. This is accomplished by determining the line between the extreme points of the three coordinate pairs: slope, m, and intercept, i, and elevation of the line at the middle station, y_c, are

$$\begin{gathered} m = (y_3-y_1)/(x_3-x_1), \quad i = y_1 - mx_1, \\ Y_c = mx_2 + i \end{gathered} \tag{4}$$

If $y_2 > y_C$, then R is set to be negative.

QUOTE

On a cursory look nothing much positive emerges when a survey is made of the relatively little written about religion by the most prominent figures of twentieth-century science. Planck, with his groping for a personal God, still belonged to an older school, which, however, shied away from historical revelation. Bohr’s views on religion were those of Harald Hoffding, the Danish forerunner of William James. They amounted to the recognition of some purely natural aspirations in man complementing sheer rationality. In Schrodinger’s Buddhism there was no room for a transcendental, personal God, let alone for His stepping into history through a specific revelation. The ***Physics and Beyond*** of Heisenberg contains no metaphysics worthy of that name. Its concluding note, the enthrallment of a Beethoven trio, is certainly beyond physics, but not at all beyond ***physis*** or nature. Pascal’s fervent commitment to the God of Abraham, Isaac, and Jacob, revealing himself in Jesus, had no appeal for Heisenberg .

Others, like De Broglie and Dirac, kept a studied silence about religion, in accordance with the widely shared view that science alone is public knowledge, or a knowledge with objective validity, whereas religion is merely a private knowledge, that is, a respectable personal opinion at best.

Jaki, Stanley L. 1986. ***Chance or Reality and Other Essays.*** University Press of America. Lanham, MD p. 163.